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THE MATHEMATICS TEACHER

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THE MATHEMATICS TEACHER

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THE MATHEMATICS TEACHER

Volume XL Number 8

Edited by William David Reeve

Enrichment of Mathematics Instruction*

By PAUL L. TRUMP
University of Wisconsin

IN CONTEMPLATING the message to bring you on the subject as assigned, I must admit a degree of indecision. No speaker could be accorded the honor of greater freedom than the subject appears to allow. As a check on my thinking I referred to that ever useful refuge, the dictionary. There I found among the various definitions of *enrich*, two which suggested the importance of a basic distinction. One definition implied to me the artificial type of fads and frills by which we are sometimes intrigued away from fundamental emphases in mathematics. The other impressed me as providing a much more substantial basis for accomplishing the end results we really seek. The first of the two definitions of enrich was "To supply with ornament, to adorn." The second was "To add to the endowments of, to increase the knowledge or capacities of, to impart a richer quality to."

There is a type of enrichment in mathematics teaching with which we become concerned which is too often superficial. Sometimes, perhaps the end *does* justify the means. If by any device we can arouse excitement, interest, pleasure and attention, we have at least partial requisites for effective learning. We must always be on the alert, however, that the end be a significant one and not a false evidence of mathematical advancement.

It is important for us to recognize that we are concerned with the enrichment of *mathematics instruction*, not the enrichment of *mathematics*. The beauty of a sunrise is none the less perfect if one is at the moment fast asleep. It is not our problem to "adorn" a subject which is intrinsically lifeless and devoid of beauty. It *is* our problem to provide boys and girls with the kind of experiences which will enable them to discover the beauty which is there. Probably the first and prime demand that this fact makes upon you and me as teachers, is our *own* awareness of that richness and beauty. An inspired teacher comes at the top of my list of prerequisites for *rich* teaching.

You have heard that we teach children, not subjects. I submit that this argument often degenerates to a splendid example of the "either-or" type of careless reasoning. We must know our pupils *and* we must know our subject. We have all been concerned about the teacher who knows mathematics but fails to recognize the individuality of children and how they learn.

* Read before the Annual Spring Conference —Mathematics Teachers of Minnesota, April 25, 1947.

In many such instances, I would suggest. that such a teacher often does not in fact really know the subject. One does not come to know the Rocky Mountains as a result of a travelogue, even though he may be able to name every peak, valley, lake and waterfall. A painting of a scene in the Rockies implies much more to one who has been there than to one who has never had such an experience.

Many of us have learned nursery rhymes in a strange language. We may derive a sense of pleasure from the rhyme and rhythm. It might be that knowing the language would add little else. Such would not be true of real poetry. It is both the thought expressed and the mode of expression of that thought which lends richness to poetry. The thought expressed cannot stir the reader, however, if his experience does not enable him to identify himself with the poet. Mathematics is much more than a series of definitions, rules, and techniques. The teacher who really knows mathematics will be seldom long guilty of not teaching children.

We have all been concerned about the teacher who knows children but is deficient in the knowledge and appreciation of mathematics. This is often the teacher who would feel that mathematics needs adornment, and who would be guilty of dependence upon the artificial tricks of making mathematics "practical." It is as though the enthusiasm of a child were aroused in anticipation of a large, attractive ice cream sundae only to find that it was entirely froth and tastelessness.

Both children and mathematics must be known and appreciated in order to guide children toward the real values that lie in the study of mathematics. If a choice must be made, I submit that a competent individual can discover children in a classroom equipped with children and a textbook much more readily than she can thus discover mathematics.

Boys and girls enjoy performing the techniques and applying the rules of mathematics if they have reasonable success. Manipulative puzzles are fun if you are challenged, but not completely frustrated. There are many ways in which we as teachers can assist pupils to achieve this success in the forms of mathematics. You have, perhaps, been much intrigued by various devices in teaching whose purpose was to elicit a correct response pattern without regard to its reall significance, meaning, or mathematical basis. If the result desired is only to enable the child to exhibit a better batting average in substituting numbers in a formula, or in setting up the equations for a particular type of verbal problem, or to become arbitrarily aware of the fact that the hypothesis must occur somewhere in the proof of a geometric theorem, then the device cannot be called enrichment.

One difficulty we have is that most of our testing of mathematical achievement is based upon evidence that the child knows the rules and can perform the skills of mathematical manipulation. One who understands mathematical thinking can do these things, but the converse does not apply. We might as well reason "this animal is a dog because it has four legs." Even the pupil who expects to become an engineer or scientist is done an injustice if we provide him with only the tool values of mathematics without sufficient insight for the proper use of these tools. Whatever tool value the achieved skills have is minimized by his handicap. In addition, the forgetting curve suffers an unnecessarily sharp decline. Possibly some lowering of standards in terms of the number, complexity, and abstractness of the formalisms of mathematics and a raising of standards in the direction of understanding based upon meaningful experience and realistic problem solving would result in a residue of more skills and a greater proficiency in their use. This is something that cannot be achieved without careful educational research. It calls for the development of testing procedures far beyond those now in common use. It calls for professionally alert and active mathematics teachers.

It probably demands the lightening of teaching loads and provision for programs of research in service. It calls for teachers with the purpose and the time and energy to learn and profit by what others are doing, to put sound recommendations into practice, to contribute to the ever present problem of sounder and more effective educational practice. Increased salaries will aid in attracting competent young men and women into the profession. They cannot do much to increase the professional accomplishment of those who teach in spite of an inadequate remuneration, unless it be by removing the pressure of necessity for supplementing income by summer employment rather than professional activity. A teacher's profession is a year-round, full time job.

I firmly believe that true enrichment will result in greater success and retention in the formalisms of mathematics. The converse does not follow. Skill achieved in the techniques alone cannot insure that our teaching has been rich.

One of the primary essentials for the enrichment of instruction in mathematics is realism as to our purposes. Mathematics teachers guilty of wishful thinking are Exhibit A in the case against transfer of training. This applies to all types of purposes from proficiency in arithmetic processes to the more intangible objectives of critical thinking and honesty. Minimal proficiency in basic mathematical skills at a *given time* is not enough. This proficiency must be maintained and must be on tap when the opportunity for use presents itself. The measure of success of a school program must be in terms of the behaviors of its graduates. Two solutions for this problem are commonly suggested. One is the increased emphasis on meaning rather than complete dependence upon drill for the fixing of skills. The other is continuity in school curricula and a realistic program of maintenance and evaluation. Although I cannot subscribe to all that has been proposed in the name of the meaning approach in teaching arithmetic, I believe the approach is essentially sound. In the hands of an inadequately trained teacher, it may easily degenerate into the old drill and reliance upon the automatic meaningless response type of rote learning. You know people who lack the ability to appreciate symphonic music but who nevertheless know what one properly says who has really enjoyed such a performance.

In order that algebra instruction be rich, it must be built upon a rich foundation in arithmetic instruction. This is not to say that the algebra teacher is warranted in excusing herself merely because the child's arithmetic teachers may have fallen short of the mark. No teacher of mathematics can avoid responsibility on such grounds. One major purpose in the teaching of algebra must be added enrichment of the arithmetic background. This provides a sounder basis for algebra and insures that the child's facility in arithmetic and fundamental abilities in quantitative thinking are maintained and strengthened.

The teacher of sixth grade arithmetic has certain major and specific responsibilities in a school curriculum. The child is building upon what he has developed as a result of his previous eleven or twelve years of experience. It may be that you are teaching under a course of study which has an educationally sound time schedule of topics for the group. By educationally sound, I mean in terms of group readiness and maturity level. At the same time, the sixth grade teacher is responsible for her contribution to many continuing objectives. It is often in the area of such continuing objectives that the real chance for enrichment presents itself.

Let us consider, for example, the general concept of measurement. There is merit in isolating the various skills and concepts involved and placing them at appropriate points in the course of study. The basic problems of accuracy and precision, of computation with approximate numbers which arise from measurement as used, for example in indirect measurements of area

are not problems which can be assigned to one teacher or grade level. The arithmetic, the algebra, the geometry and the trigonometry teachers each have a contribution to make toward building a rich understanding and proficiency in the concepts of direct and indirect measurement and approximate computation. Each must have knowledge of the desired end result. Each must know where the child is when he passes into her hands, each must realize that where the child is cannot be determined by glancing at the topical index of a textbook he has used, each must capitalize on the experiences the child has had in other classrooms and outside the school, each must be conscious of purposes extending far beyond the computational and manipulative skills for which she has specific responsibility. Nothing can replace experience with actual problems involving the child's activity in taking measurements, in estimating measurements and then checking the estimates, of facing a problem involving measurement and seeking to discover the data that must be obtained, how to obtain it, what precision is necessary, how to organize the data and the computations, and how to judge the accuracy of the results. I do not say that all work in the mathematics classroom must be of the laboratory project type. There must be some provision for such types of well planned activity to inspire meaning, purpose and richness in the traditional type of classroom activity.

Problems of measurement are the source of need for much that is taught children in our classes, and yet we find that even though achievement as measured by our tests has been satisfactory, children show up later with lack of ability to use these skills when confronted by such problems. We have sought to remedy these difficulties by so-called applied problems used *after* a degree of mastery is attained. The applications should often be used as bases of learning in the presentation stage *before* drill. Drill is more effective if the process being drilled upon is meaningful. It is more difficult to superimpose meaning on a mechanical process once drilled upon than it is to begin with a meaning emphasis and then by drill fix the responses found useful.

One of the objectives in algebra instruction which has received emphasis in the past few years is tagged with the label relational thinking. We have been so impressed with its importance that we have developed teaching units. Textbook writers have inserted units on variation and even scattered maintenance examples in occasional spots. Teachers report that they have difficulty with these units.

The emphasis on relational thinking as an objective in teaching algebra was an attempt to enrich algebra. It was an attempt to breathe life and meaning into formulas and algebraic expressions. There are two major explanations for the relative ineffectiveness of this attempt. One is that even such materials are often made formal and the object of rote learning. The other is that we're speaking of a purpose which cannot be achieved by an occasional emphasis, but only by an all-pervading point of view in teaching mathematics. It is a point of view that must become a part of all instruction whenever we are considering quantitative elements. The entire purpose is lost if "relational thinking" becomes an isolated topic added to the regular curriculum with no apparent relationship existing between it and other topics. In the arithmetic class, when the child is being taught percentage he can see that doubling the rate has a definite describable quantitative effect upon the percentage. In fact, this kind of emphasis used in the early presentation would help the child to understand what a rate per cent really is. If each ticket sold to a school entertainment means a ten cent profit to the child's class fund, he will have no trouble in telling you what increase in profit accompanies an increase of 50 tickets in the number sold. If the base increases by 50 and the rate is 10%, what is the increase in the percentage? While these problems

are equivalent, the child thinks of the first as common sense and the second as a difficult problem in percentage! If there are two hundred pupils in school, each added absence means an increase of one-half in the per cent of absences. If you increase the base by a given amount, what is the change in the percentage? These are the kinds of observations which the child who is doing something more than busy work will be making if he is given the opportunity. We pass over many of these opportunities and then find that when we try to formalize and abstract such in a unit on variation, we have not achieved enrichment. We have merely added more of the hard crust of formal response.

How mature must a child be to develop the idea that if his family takes 90 quarts of milk a month, then each time the price increases by one cent per quart, the monthly milk bill increases by 90 cents? Try it out on your class. This can be done long before any concept of literal number, variable, or formula has been formally presented. It should help in developing an understanding of what a formula is. Eventually it can be abstracted and expressed in mathematical vocabulary and symbolism.

There are many opportunities to use this point of view in solving verbal problems. It leads to such questions as "What is the effect upon the solution of this problem if one of the datum be changed?" For example, consider the problem of what proportions of 30 cent coffee and 50 cent coffee should be mixed to obtain 40 cent coffee. The child may answer "Mix them half and half." In some classrooms, he would get his ears pinned back for guessing and not going through the routine. However, here is a beautiful starting point for some good relational thinking. Before any attempt is made to achieve an algebraic solution. the child can sometimes answer more searching questions than he is able to answer after the formal solution has been presented. How will the result be changed if more expensive coffee replaces the 50 cent coffee? Suppose you replace the 50 cent coffee with cheaper coffee; with 35 cent coffee? At this stage, he can see that it is ridiculous mathematically (although it may be good business) to try to make 40 cent coffee by mixing portions of 30 and 35 cent coffee. Later he will be interested to learn how this difficulty shows up in attempting the algebraic solution.

As a fitting climax, it would be instructive to solve the problem of mixing milk at m cents a quart and cream at c cents to obtain rich milk at r cents. The formula becomes $c-r/c-m$ quarts of milk to $r-m/c-m$ quarts of cream. Then an interpretation of the questions asked initially can be made in terms of these algebraic expressions. For example, suppose $m=c$; suppose m and c each are less than r? Suppose $c>m$, what is the effect of increasing r?; of increasing m? What is the sum of the two expressions? Suppose $m=o$. If the mixing problem is made real, much of the algebra interpreted in terms of it can be made real. We could well afford to give more attention to such procedures in preference to undue concern about whether or not this child will ever have to mix coffee in a grocery store.

No apologies need be made for a course in algebra which does a good job in accomplishing such purposes even though some of the more formal work in factoring or complex fractions should have to be slighted. I believe we would have more success with the formal skills of algebra if we spent a greater proportion of our time on algebra as an aid to thinking. We can gain little by extending meaningful processes too rapidly and too far beyond the meaningful stage. For most of our pupils we would do well to strive for more significant mastery of fewer ideas.

This same point of view of interrelationship and variation should carry over into the geometry classroom. How do the base angles of an isosceles triangle change as the base increases in length while the equal sides remain unchanged? How does an

angle change if its vertex passes along a fixed circle through two points through which the sides of the angle pass? Visualize the change in the angle bisector, the altitude, and median as a triangle changes shape. The pupil should be constantly encouraged to visualize geometric figures in motion, to observe fixed or invariant relations and relative changes affected when other changes are accomplished. Models are helpful, and by using rubber bands or elastic cord many instructive models can be planned and built by the pupils. A particularly useful but simple model is that of a parallelogram whose sides are pivoted. One or two diagonals in the form of rubber bands will enable the pupil to watch pairs of triangles change but remain congruent. The diagonals change in length but they continue to bisect each other. If such concepts of freedom of motion under the hypotheses were developed, the child's ability to understand a theorem, or analyse and discover a proof would be enhanced.

The efforts to enrich mathematics teaching have involved statements of purposes concerning many other types of thinking behavior. It is the subject of plane geometry which carries the heavy load in much of this work. There has been a promising swing to specific emphasis on the method of thinking utilized in demonstrative geometry as apart from the geometric facts embodied in its theorems. We have discovered that tracking through the thought processes involved in a geometric proof need have little or no effect upon the thinking behaviors of children. The child needs to see the reality of the dual emphasis upon proof and upon geometric facts and relationships. The effort to change thinking behaviors must be specific and realistic. The child must understand what kind of a game is being played and must be encouraged in the development of his own powers of initiative and creativeness. I would prefer the submission of a less polished proof which was the product of the child's constructive thought to his presentation, even word perfect, of a book proof.

With the inclusion in the junior high school curriculum of increasing amounts of informal geometry, we have an opportunity to do something about the inadequate results which are obtained in many of our geometry classes. The beginning point in geometry as a logical organization of subject matter is the assumptions and definitions upon which geometry is based. This provides a very difficult start from a psychological point of view. The child has observed geometrical elements about him. If we were willing to strip geometry of its logical structure, we could probably do a much more effective job in less time through an experimental, inductive approach. However, I firmly believe that much value can be realized by properly taking advantage of the extent to which geometry lends itself to postulational thinking and proof. We must strive specifically for such value, and not leave it to chance. I do not believe in an abstract course in logic for high school pupils. Thinking can be formalized. That fact presents a real danger signal in our attempt to develop the nature of proof type of emphasis in teaching geometry.

In another paper published recently,[1] I gave suggestions on the utilizing of pupil experiences in their discovery of mathematics. Specific suggestions were made, particularly for the enrichment of geometry teaching.

If our teaching is to be rich, our own experience with children and with mathematics must be rich. The richness of mathematics lies not in its formalisms and skills but in mathematics as a language for expressing relations between quantitative measures. Many teaching techniques are directed at getting results in the form of ability to set up or solve a

[1] Trump, Paul L., "Utilizing Pupil Experiences in Their Discovery of Mathematics," *School Science and Mathematics*, vol. 46, #6 (June 1946), pp. 521–527.

particular type of equation or to perform correctly a certain mathematical manipulation. The secret of enrichment lies much more deeply than that. I believe it lies primarily in two directions. In the direction of meaningfulness through the proper utilizations of pupil experiences and activity in meaningful learning, and in the direction of efficiently directed experience toward the real underlying purposes of mathematics as a way of thinking.

Progress in mathematics instruction can be realized only through alert professional work on the part of classroom teachers. Your presence here today is an evidence of your professional interest. I hope you may have been able to fasten upon at least one suggestive idea during the time which you have given me. However, such occasions as these cannot accomplish the job. That can only be accomplished in your classrooms and in your solutions of problems as you face them. Your thinking, reading, and study will build within you a point of view, a specificity of purpose, a standard of teaching excellence which will be the conditioning factor in your meeting the every-day problems of the classroom, There is one reward I can promise you for lots of hard work. That is the thrill and satisfaction that only a professionally active teacher can know. Teaching algebra one year many times can be deadening and boring. Teaching boys and girls is more exciting. But the ever enlarging horizons of purpose and attainment to a teacher who is alert to professional problems, who is able to inspire initiative and confidence in boys and girls, who is confident that they are learning something worth while which will reflect in their behavior long after they leave her classroom wherever they go, is the extremely satisfying reward of the master teacher.

I cannot refrain, at this point, from inserting a challenge to mathematics departments and departments of education in our teacher training institutions to recognize more fully and realistically their responsibility in the preparation of teachers who are equipped to make the most of their opportunities in teaching mathematics.

Perhaps some of my suggestions today would not be entirely accepted by all whom I would be willing to classify as master teachers. Some of these disagreements would be evidence of the need for experimentation and educational research. I dare to hope that some of that research will take place in your classrooms.

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The Necessity of Considering the Social Phase of Instruction in Mathematics

By Leo J. Brueckner
University of Minnesota, Minneapolis, Minn.

The impact of the war and its aftermath has been particularly heavy on instruction in mathematics. Criticisms of the work of the schools in this field have been sharp and direct due to the fact that both industry and the armed forces found that the mathematical background of many individuals was decidedly weak and in most cases very limited. The result was that not only did industry and armed forces establish intensive basic training programs in so-called mathematical essentials but also that the schools and colleges of the country began to examine their courses in the various branches of mathematics systematically and critically to determine their strengths and weaknesses. As a consequence of these activities there have been important developments in both methods and materials of instruction that are of great significance for those concerned with the improvement of the work in mathematics in our schools.

The war made it crystal clear that the affairs of our modern technological society require a much wider use of the technics of the various branches of mathematics than the activities of our earlier rural agricultural social order. It seems evident however that the same mathematics that is needed to build war ships, tanks, guns, and trucks for war purposes is also needed to build airplanes, tractors, and precision instruments for peacetime use. The same mathematical skills needed to navigate a bomber or to operate many of the instruments of precision needed in war are also needed in similar activities in peacetime. There is a multitude of war time activities adjusted and expanded to meet peace time needs in which considerable skill in the use of mathematical technics is necessary. Such practices as rent control, sugar rationing, priorities, and the use of index numbers as a basis of prediction and control are managed only through the technics of mathematics, and these practices have continued after the close of the war. It should thus be clear that the substantial expansion of mathematical instruction should not be regarded as only a wartime measure but rather as a development that will be continued and even extended after the war. Changes in our courses growing out of the demands of the war that emphasize the practical applications of mathematical technics in human affairs should be carefully examined with full recognition of their functions in the long range development of the curriculum in mathematics.

Because of the demonstrated crucial nature of the work in mathematics this subject obviously should be regarded as an essential element of the core curriculum of general education. The trend in enrollments in mathematics in the years immediately preceding the war was definitely downward and large numbers of students were given no training whatsoever in this important area. There is evidence that since 1942 there has been a change in the direction of this trend and that there has been a marked increase in the enrollment in all branches of mathematics. In some high schools there were offered during the war such narrow specialized courses as "Mathematics for Nurses," and "Mathematics for Navigation." It is generally believed that this is an unwise and ineffective approach to the problem of curriculum development and that a sounder procedure would be to integrate this work into the regular courses and thus to enrich their content by more emphasis on up-to-date applications of mathematics chosen

from a variety of fields. Each fundamental mathematical principle being studied can easily be illustrated by situations taken from industry, the armed services, and home front activities. In this way the pupils will learn to see the broad application of a basic idea and be able to recall it whenever they need it even though the principle may arise in a novel situation. The major task involved in this program is to make the fundamental courses thoroughly functional through emphasis on essentials and through the use of many highly practical illustrations, selected from normal and wartime activities.

Mathematics should deal with the study of the social phase of the subject as well as with its technical aspects. Attainment of the objectives of instruction in mathematics is possible only if meaning, the fact that the pupils see sense and utility in what they learn, is made the central consideration at all stages. The desired outcomes are most likely to be achieved by making certain that the pupils have concrete experiences in which they can see how number functions in life situations that are vital and significant to them, and if at the same time the teacher takes the steps needed to make sure that the mathematical meaning of the quantitative elements involved is grasped. The teacher should plan definitely and systematically to develop quantitative thinking by making the pupils sensitive to the ways in which number functions in the affairs of daily life and by trying to develop insight into the mathematical procedures through experiences that give the learner practice in dealing with his problems systematically and effectively. Such an approach will tend to reduce the abstractness and formal nature of much of the instruction in mathematics and to make it a field of learning that will have a wide appeal to the pupils. The important report, *Mathematics in General Education*, pointed out that the basis of the mathematics curriculum should be the study of significant social problems of concern to the pupils and the community. The formulation of these problems so as to make them definite and meaningful and the consideration of the steps necessary to bring about their solution requires the use of the systematic procedures of mathematics. The approach through problem solving is especially important since it leads the pupils to see the utility and value of the use of quantitative procedures as an integral phase of the scientific method of dealing with human affairs and gives them practice in its application. The study of the processes and fundamental issues underlying such current national problems as rationing, inflation, social security, taxation, and post war planning, all of them matters of concern to all citizens, young and old, will give the pupils valuable training in the use of the kinds of procedures that are basic in the democratic process. The consideration of the quantitative aspects of these problems makes them meaningful and concrete. The approach must of course be adjusted to the level of maturity of the pupils. The purely informational aspects can be considered at almost all levels; the more technical mathematical phases can be deferred when advisable. The discussions of how these questions arose and of possible ways of solving the problems will make clear the difficulties that are faced by the nation in dealing with them. The bearing of these problems on the lives of the students not only in war time but also in times of peace can thus be made clearly evident to them and their own responsibility for taking an active part in their solution can be amply demonstrated. The point of view here expressed is well stated in the following quotation:

> . . . The fruits of scientific (and mathematical) thinking in terms of application to the problems of human living require democratic societies for their full development. More important still, the kind of scientific and mathematical thinking that the Committee advocates developing in students—that is, thinking applied to the real problems in the basic relationships of living—provides a valuable tool for the preservation of a democratic way of life. Students who are in

the habit of formulating real problems, and of insisting on genuine solutions, who know how to judge, collect, and interpret data, who are not misled by inaccurate or misleading statistics, and who know how to recognize valid proof, will not so easily be misled by propaganda, suppression of evidence, systematic calumny, demagoguery, or mystical symbols.[1]

The contents of organized courses in mathematics at all grade levels should be determined by the analysis of the mathematical procedures that are needed by the pupils and the community in the study and solution of these vital social problems. If this is done we can be certain that the subject matter being considered will be mathematically meaningful and at the same time socially significant. The procedure to be used in curriculum making from this point of view is well illustrated by the recent report of a committee of the National Council of Teachers of Mathematics on "Essential Mathematics for Meeting War Needs." The work of this committee grew out of the problem of determining the nature and scope of the courses that should be offered by the schools to meet the needs of those who were about to enter the armed forces and also the amount and nature of mathematics that should in the future be required of all pupils. Should all students be obliged to take the conventional courses in mathematics including algebra, geometry, trigonometry, and so on, which as now organized deal with very few topics related to the war or current social problems? Should new courses be established, based entirely on social needs, or should the old courses be modified? The report of the committee based on a comprehensive study of the basic mathematical requirements of the armed forces as revealed by direct study of their activities shows that there are certain skills and abilities that are almost universally demanded but that these needs are not nearly as sweeping nor as difficult as the subject matter included in the conventional present day courses in mathematics. The scope of these courses can therefore be greatly reduced and still meet the needs of the armed forces. The applications of the various branches of mathematics can at the same time be greatly enriched by relating their content more closely to the war time uses of mathematics as revealed by this committee report.

It would of course not be desirable in the future to make the sole basis of determining the contents of courses in mathematics the needs of the armed forces. What is clearly needed is a comprehensive systematic study by competent professional groups of the mathematical procedures that are needed in all important social activities, particularly those that are required to deal effectively with the problems that arise in the social, economic, political, and occupational affairs of community life. It seems clear that in the future there must be a general reconstruction of the curriculum in mathematics at the high school level if the subject is to be made functional and if mathematics is to make the contribution it should make to the management of the affairs of a democratic society. This approach will not result in the doing away with mathematics as such in the curriculum. The results will rather be the development of a program of instruction in this area that will be so vital and important and so well arranged that all students regardless of ability will be given instruction in the basic essentials, adapted to their potentialities and future destinies. Furthermore there is every reason why the problems with mathematical relations that arise in the various curriculum areas other than mathematics, for example, science, health, social studies, vocational areas, and even art, should be dealt with in connection with the work in the fields involved, possibly through the introduction of well devised units dealing with the mathematical elements regarded as essential in each area.

[1] *Mathematics in General Education.* A report of the Committee on the Function of Mathematics in General Education for the Commission on Secondary School Curriculum. D. Appleton-Century Co., New York. 1940. P. 68.

Just as we today recognize that every teacher must in fact teach the reading skills peculiar to his subject, so we should ultimately accept the point of view that every teacher should assume the responsibility for dealing with the mathematical foundations of the work in his area and that he should make the necessary mathematical applications, even though these technics are specialized and do not seem to fall within his immediate province. The values of such an approach should be thoroughly examined.

There are several important and rather practical steps that can be taken today by our high schools to meet the present situation in mathematics. In the first place, teachers of mathematics should make every effort to bring to the attention of the pupils enrolled in their courses the ways in which the subject matter being studied functions in the affairs of the world of today. These applications should deal not only with the strictly mathematical aspects of these matters but also with their broader social significance. While the teaching of the uses of mathematics by our armed forces and industry was undoubtedly a most vital and direct means of showing the utility of mathematics, the consideration of peace time problems that must be dealt with offers another valuable source of experiences in dealing with the quantitative aspects of human affairs. The longtime mathematics program should always be kept in mind in making the selection of subject matter.

Another important step for the schools to take today is to examine closely the content of what is now being taught, to deemphasize the stress that has in the past been placed on subject matter which is of little social value and hence is not of fundamental importance, and in some cases to eliminate it. In the place of this rejected material there should be added a body of content that is modern, functional, and practical. No pupil should ever be impelled by a feeling of futility or a lack of appreciation to ask the question, "Why am I required to take this course in mathematics?" The value and essential importance of his work in mathematics should be made so evident to him that he can have no doubt as to the need of mastering it. The improvement of the courses in mathematics along these lines will require the cooperative study of the problems involved by all those who are concerned with having mathematics assume its proper place in the curriculum as a vital significant field of endeavor. Workshops devoted to the study of the problems underlying this approach and to the development of the necessary instructional materials should be established in teacher training centers and school systems throughout the country. Experimental programs should also be organized to study the best ways of improving the general quality of instruction.

A third immediate step for the high schools to take is to see to it that the fundamental skills in arithmetic and other areas are maintained at a relatively high level by systematic reviews at appropriate intervals throughout the years of the high school, especially at the time when the students are about to leave school to enter some occupation or to be inducted into the armed forces. There is definite proof available that criticisms by the armed forces and industry as to the low level of arithmetical abilities of those who are leaving our schools now or who left some time in the past are justified. The evidence shows that many of our graduates are very inefficient in computational skills and that when they leave school those who have taken only one or two courses in mathematics, usually in the first two years of the high school, have almost wholly forgotten what they learned in these courses. Investigations have shown that this low level of ability is largely due to the fact that these skills deteriorate because they are not used or practiced to any extent at the high school level in courses other than mathematics. Experiments have demonstrated that a relatively short program of

systematic review and practice with suitable materials results in a quick rebuilding of these abilities to a satisfactory level. It is therefore recommended that short unit "refresher" courses in basic mathematical skills be established by high schools to provide the practice that is needed. These courses need not be more than four weeks in length. The time required to attain a satisfactory level of ability will vary from individual to individual because of differences in the variety and amount of practice required. Hence the basis of the review should be an individualized program founded on the use of systematic diagnostic tests which will enable the school to determine the needs of each student, and of well constructed practice exercises that will enable each individual to remedy his weak spots. A number of well constructed workbooks developed along these lines have been published. The review work should be limited to the essentials and should be planned in accordance with the background and future destiny of each student. Such a program of review is much more satisfactory than a blanket requirement that all students must enroll in some abstract, difficult, comprehensive course in general mathematics dealing with many technical aspects of the subject with which considerable numbers of the students have had little if any previous contact, and not organized so as to provide for individual differences. It is also recommended that the training in the more specialized technical phases of mathematics that are required by industry and business and by the various branches of the armed forces be deferred until the individual actually begins the basic training program in these lines. The refresher course to be offered by the high school should not be regarded as merely a wartime measure but as one that should be included in the postwar program.

A fourth step that has important implications is the use by the schools of the various kinds of modern aids of learning whose development has been greatly stimulated and accelerated by the activities of the training programs of the armed forces and other war agencies. The necessity of speeding up the training programs and of making them as effective as possible has led to the invention of numerous kinds of visual aids which apparently have been found to be very helpful as a means of facilitating the matsery of basic skills. The best of these materials are not yet available but a number of them have been released for general distribution. These include excellent films of various kinds demonstrating mathematical procedures and their war time applications. Many other films of this kind with generalized illustrations will undoubtedly appear in the near future. Likewise various models, exhibits, and similar concrete instructional materials have been devised to aid in the presentation of mathematical concepts and relations. Extensive use was also made of photographs, pictures, graphic representations, and a wide variety of other pictorial ways of presenting quantitative concepts and information.

Not only the armed forces but also other governmental agencies have published descriptive materials related to mathematics that are of value to teachers. This is especially true of publications of the Education Section of the War Finance Division of the United States Treasury Department which deal with the sale of war bonds and stamps, the methods of dealing with inflation, problems about taxation, and other issues that our nation faces. Other governmental agencies that publish valuable materials related to mathematics are the various planning boards, Office of Price Administration, and the various departments. At the present time there is literally a flood of such reports dealing with current issues. The teacher of mathematics will find them a valuable source of authentic information. A well selected set of these materials should be on hand in every school. Wider use should also be made of available local

community resources to give concreteness to the work in mathematics, particularly in the grades below the high school.

Selected Bibliography on Materials and Methods in Mathematics of Significance for Post War Education

I. *Curriculum Content.*

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2. Kinney, L. B. "The Reorganization of Mathematics for the Emergency." The Mathematics Teacher, 36: 3–10 (January, 1943). Discusses requirements of mathematics by armed forces, industry, and the home front and the implication for a long time program.
3. "Pre-induction Courses in Mathematics." Report of a committee setup by U. S. Office of Education. The Mathematics Teacher, 36: 114–124 (March, 1943). Contains an analysis of the basic essentials of mathematics required by the armed forces, based on an analysis of 50 instructional manuals used in training programs. The analysis suggests the basic contents of pre-induction courses.
4. "Essential Mathematics for Minimum Army Needs." A report of a committee of the National Council of Teachers of Mathematics. The Mathematics Teacher, 36: 245–283. October, 1943). Contains an analysis of mathematics needed by armed forces, based on actual field observations, direct contact, and interviews. This is the most recent report in this field and may be regarded as the most authoritative.
5. Reeve, W. D. "A Proposal for Mathematics Education in the Secondary Schools of the United States." The Mathematics Teacher, 36: 11–20 (January, 1943). Advocates a drastic elimination of non-essentials and a reorganization of the entire program in mathematics.

II. *Remedial Arithmetic in High Schools.*

1. Blair, G. M. "Remedial Arithmetic in Senior High Schools," The Mathematics Teacher, 36: 346–350. (December, 1943). Reports a survey of practices in courses in remedial arithmetic in a number of secondary schools.
2. Brueckner, L. J. "Improving Mathematical Abilities of Pre-Induction Groups." Bulletin of the National Association of Secondary School Principals, 27: 33–48 (December, 1943). Describes results of tests of arithmetic and algebra administered to high school students and men in armed forces which show a low level of ability and discusses the results of certain experiments on the effectiveness of various ways of improving instruction. Stress is placed on the necessity of using materials that provide for individual differences.

III. *Miscellaneous Materials of Instruction.*

1. Babcock, C. D., Jeffrey, E., and Troelstrup, A. Paying for the War. Bulletin No. 18, November 1942. Washington, D. C.: The National Council of the Social Studies, 1201 Sixteenth St. N.W.
2. Miller, A. B. "A Report of the Radio Committee of the National Council of Teachers of Mathematics." The Mathematics Teacher, 37: 106–109 (March, 1944). Reports survey of the use of the radio in mathematics instruction and illustrates the kinds of programs available.
3. McNemey, M. L. "Demonstration Teaching in Arithmetic by Radio." The Mathematics Teacher, 37: 110–117 (March, 1944). Discusses the use of the radio as a means of instruction and supervision of mathematics.
4. "The Teacher of Mathematics and the War Savings Program." A School's-At-War Bulletin, published by Education Section of the War Finance Division, U. S. Treasury Department, Washington 25, D. C. Also published in The Mathematics Teacher, 36: 354–375 (December, 1943). Presents a detailed analysis of what can be done by the teachers of arithmetic and mathematics to stimulate the sale of war bonds and to make this community intelligent about the use made of the funds. This division publishes many pictures, pamphlets, posters, etc., related to war finance that are available to all schools.

IV. *Selected Visual Materials Available in the Field of Mathematics.*

1. A Thousand Hours. 10 min. 16-sd-$36.50; rent $1.50, 1942. Jam Handy. Opens with a typical neighborhood group of boys who have their own shop in a garage where they "experiment" and build. The captain of one of the great airline transports is a good friend of theirs and in the course of the picture inspires the boys with an interesting explanation of the background of the sciences necessary to pilot training. He shows the boys

how the "radio beam" operates to guide planes in the air. As the picture ends, the boy from the group who wants to be a pilot had decided that "such unnecessary stuff as mathematics" is essential to him if he expects to realize his ambition to become a pilot.

2. GEOMETRY IN ACTION. (Geometry brought to life series) 10 min. 16-sd-$30; rent $1.50. 1940. Bald Eagle.
3. RECTILINEAR COORDINATES. (Knowledge Builders series) 10 min. 16-sd-$40; rent $1.25. This film shows how a point moves and becomes a line. After explaining positive and negative numbers from coordinate positions, it branches out into tridimensional areas. An excellent film in many ways, although technically scarcely more than average. A noble film attempt in the comparatively "Unfilmed" field of mathematics. Only suitable for advanced students. Should be useful for solid geometry, advanced math, physics.
4. ELEMENTS OF ELECTRICAL CIRCUITS. (1 reel) $1.00. Reveals the nature of electric currents and circuits. Animated drawing and photographic demonstrations explain electron motions, conductors, insulators, and factors affecting resistance. Detail explanation of resistance and the ohm, rate of flow and the ampere electromotive force and the volt lead to an understanding of their interrelationship as expressed in Ohm's Law. (Erpi).
5. RECEIVING RADIO MESSAGES. (1 reel.) $1.00. Explains in detail by animated drawing the fundamental steps and principles in radio reception by using the simplest form of receiver—the crystal set. The film explains aerial-ground capacity, resonance, tuning, detection, rectification, the crystal, earphones, demodulation, fixed condenser, and symbols used by technicians. (Erpi.)
6. THREE-DIMENSIONAL VECTOGRAPH PICTURES AND SLIDES. Very soon there will be available three-dimensional vectograph pictures and slides which, viewed through inexpensive Polaroid glasses, produce a realistic three-dimensional effect. A series on geometry will be one of the first released. These will be available from Society for Visual Education, Inc., 100 East Ohio Street, Chicago, Illinois.

V. *Workbooks and Practice Materials.*

Almost all publishers of instructional materials have published a wide variety of workbooks and textbooks intended to meet the needs of "refresher courses." These should be examined in all cases to determine the extent to which they deal with functional content and are organized to provide for individual differences as defined in the preceding discussion.

The Annual Meeting of the National Council of Teachers of Mathematics will be held in Indianapolis, Indiana, on April 2 and 3, 1948. Headquarters will be at the Hotel Claypool. Better secure your hotel reservations now. A complete program will appear in a later issue of The Mathematics Teacher.

Why not enter a year's subscription to The Mathematics Teacher for some friend as a Xmas present for 1948?—Editor

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Will the readers of "The Mathematics Teacher" please order the Yearbooks of The National Council of Teachers of Mathematics directly from "The Bureau of Publications," Teachers College, Columbia University, 525 West 120th St., New York 27, N.Y. Do not send orders or money to "The Mathematics Teacher" as this will only delay your receiving the Yearbooks, and will cause us a great deal of trouble and unnecessary expense.—Editor

Interpretation of "College Preparation" by Individual Teachers of High School Mathematics*

By H. C. TRIMBLE
Florida State University, Tallahassee, Fla.

THE IDEA that pressure from the colleges has been a serious obstacle in the way of curriculum reform in the high school is a familiar one. Last spring I had an opportunity to visit a representative sample of Iowa high schools. Because I am employed in college teaching, and because I have heard so much about college domination of high school curriculum, I kept looking for evidences of the influence of the college in shaping the thinking of high school people.

The comments which follow are concerned only with the reactions of individual teachers of high school mathematics. The much broader question of the total interaction of the college and the high school is left for curriculum experts.

I want to make a few remarks based upon my own observations and then to suggest some questions which I have tried to answer for myself. It is my hope that you too will seek answers to these questions and not that you will accept my answers.

I want to mention three convictions which many teachers of high school mathematics seem to have acquired as they endeavoured to interpret the wishes of the colleges. The first of these is summarized by the phrase, "certain mathematical topics are essential for the college preparatory student"; the second, "the fields of arithmetic, algebra, and geometry must be taught separately"; the third, "standardized tests issued by colleges are a valid measure of a student's preparation for college mathematics."

From the point of view of the high school teacher the evidence for the first conviction is compelling. Although some colleges have "let down standards" to admit students who have not had certian standard courses, there are still colleges which demand these courses for entrance. Moreover the college catalogues rather generally advise a certain number of units of mathematics in high school. When questions relative to the content of these courses are directed to college departments of mathematics the answers are characteristically given in terms of topics. To use algebra as an example, it should include factoring, algebraic fractions, quadratics and the like. This seems to the high school teacher to imply that these topics should be covered in somewhat the form in which they are presented in a reputable high school algebra text. This chain of reasoning—required courses, to topical outlines, to accepted text materials —seems to justify the conviction that certain topics must be studied in a certain form by the college preparatory student.

Several rather specific examples of the effect of this first conviction might be cited. I have found some high school teachers using one text by preference to another text because they were not sure that the second text was really an algebra book. Although they found it to be more readable, they feared the trap of making things too easy for the students who would go on to college. The first text presented algebra which they were sure was algebra. The texts which took up the accepted topics without too much "padding" were the safe texts.

Even more important than the selection of a text is the manner in which the text, once chosen, is used. Though they have been told in methods courses to fit the subject matter to the individual student

* Read before the Mathematics Section of the Iowa Academy of Science, Cedar Falls, April 19, 1947.

or class, many high school teachers of mathematics follow their text slavishly. Even if the preface of the text itself makes certain suggestions for providing for differences among classes and among boys and girls within a class, there are a great many teachers who feel insecure about skipping any of the topics presented in the text. The conviction that these topics are essential for college preparation is to a large degree responsible for this insecurity. It is not uncommon to find an algebra teacher who plans her work for a year in advance by deciding how many pages of the text must be covered in each recitation and then following this plan methodically throughout the year. Even when it becomes clear that a class has lost its way and needs a fresh approach to a topic, the need to cover the text is often permitted to stand in the way of remedial instruction suited to the level of development of the class.

The second conviction, that arithmetic, algebra, and geometry must be taught separately, is also based upon the teacher's interpretation of the wishes of the colleges. Again, a perusal of college catalogues or a talk with a representative of a college department of mathematics seems to bear this out. Although the colleges have learned from bitter experience that a course in general mathematics may mean anything or nothing, they still seem to cling to the notion that a course in geometry does, or should, or at least must mean something. Hence the colleges and members of their mathematics departments talk to teachers in terms of courses in arithmetic, algebra, and geometry.

Many teachers who are willing to adopt general mathematics for students who seem certain to stay clear of colleges are sure that courses in general mathematics have no value for, and may perhaps be quite harmful to, any student who may someday face a college registrar. Some teachers go further to look with suspicion upon the efforts of some students of the mathematics curriculum who suggest activities to be carried on in the geometry class which are designed to maintain and extend skills of algebra. They infer from the sayings and actions of college people that geometry should be geometry. Any attempt to water down geometry with related work in arithmetic and algebra represents a lowering of standards which may be permissible for the non-college student but is to be avoided for college preparatory students.

Even some of the teachers who maintain refreshingly critical attitudes in connection with most issues, break down completely in the presence of standardized test scores. Here at least, these teachers feel, they are on safe ground. Good performance of their students in comparison to national or state norms surely means relatively good preparation for college. This third conviction is firmly held especially by teachers whose students rate well in comparison with the published norms.

The acceptance of standardized tests as valid measures of students' preparation for college mathematics has very far reaching effects. Such acceptance is closely related to the previously mentioned convictions. For how can a teacher better assure his students of good test ratings than by covering all of the topics in the text? Moreover what better way is there to prepare for a carefully purified examination in geometry than to study carefully purified geometry?

Not only do standardized tests encourage teachers to think in terms of topics and type problems on each topic, and of arithmetic, algebra, and geometry each in a beautifully purified state, but they have still further effect in rendering many teachers of high school mathematics incapable of rational thought about the problems involved in teaching their subject effectively.

Every report by a national committee on the curriculum in high school mathematics has stated objectives which are not measured by existing standardized tests. The authors of the most widely used tests

frequently admit this and sometimes use italics to emphasize the limitations of their tests. Yet many teachers are overwhelmed by the numbers on the test booklets and summary sheets to a degree which makes such precautions useless. Other objectives fade—ability to do type problems often chosen on the basis of their statistical contribution to the total score on the test, becomes the sole objective in the mathematics classroom. All this happens as the teacher endeavours to give the college what she has come to believe the college wants.

The questions which this raises in my mind can be reduced to three. (1) What *do* we in the colleges want? (2) How can we make our wants known to teachers of high school mathematics? (3) What assurance can we give to high school teachers that their students will get a fair chance to show what they know as they enter college?

My own answers to these questions have been born in pain. They represent not what I wish were true in a world different from that in which we live but rather the nearest approach to my ideals which I believe can be realized soon. Without wishing to cause you any of the pain I've felt myself I want to urge you to seek your own answers to these questions.

I think colleges want as much training in mathematics as the high school can present in a meaningful way to its students. I know that I do not personally want facility in algebraic manipulation if such facility is achieved as a set of conditioned reflexes, even if these reflexes are sufficiently differentiated to produce scores at the 90th percentile on a standardized test in algebra. I want students coming up to college with an understanding of number relationships and some facility in expressing and exploiting these relationships with mathematical tools. I want students who know that quantities are compared by difference and also by ratio, that each method has peculiar advantages, and that the comparisons may often be helpfully expressed in equations, graphs and tables. Whether this knowledge is gained in studying arithmetic, algebra, or geometry is not a matter of primary concern to me.

A complete listing of what I want as a college teacher would be lengthy and might lead me to stray from the point. In a word, I want as many big ideas as the entering student can carry, not a bag of tricks tarnished as a result of neglect.

In answer to the second question, I believe that the time is overdue when college people should cease to list topics and begin to state big ideas in answer to the persistent question, "What should we teach the college preparatory student?" The high school teacher will need specific statements of the big ideas. The effort to state what we want in an unambiguous way will lead us to a healthy examination of the importance of the topics which we in colleges cherish most. It will mean work and soul searching. But it may in the end save for our subject its due place in the high school curriculum.

I would like to quote a few passages from a recent article on mathematics curriculum. From the "Second Report of the Commission on Post-War Plans," THE MATHEMATICS TEACHER for May 1945:

"Thesis 10. The mathematics for grades 7 and 8 should be planned as a unified program and should be built around a few broad categories.—The program should be organized around (1) Number and computation, (2) The geometry of everyday life, (3) Graphic representation, (4) An introduction to the essentials of elementary algebra (formula and equation).

Thesis 13. In most schools first year algebra should be evaluated in terms of good practice.—Today good teachers of algebra—(16) recognize that it is far better to teach a few concepts well than to teach many concepts superficially.

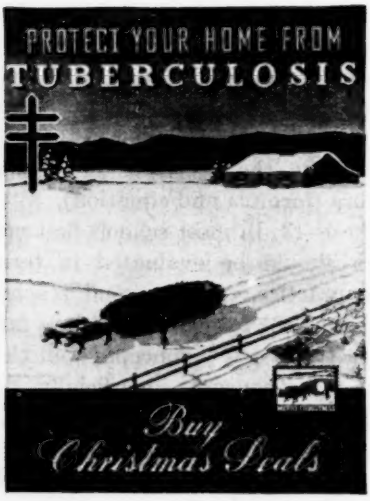
Thesis 17. The work of each year (10 to 12) should be organized into a few large units built around key concepts and fundamental principles."

Although Thesis 10 does list four broad

categories, I think I may fairly say that even the commission on post war plans has as yet neglected to list the "key concepts" and "fundamental principles" around which the high school curriculum should be built. Unless the colleges can do better than this in stating what they want, I believe the high school teachers are justified in their impression that the colleges want topic coverage and compartmentalized mathematics. Good practice may be defined in general terms as in Thesis 13, but when it comes right down to a detailed program the colleges and commissions still talk about subjects and topics.

It is my own belief that the colleges should cease to list courses and topics within these courses as the preparation they want the high schools to provide. I believe the colleges should emphasize rather the big ideas from mathematics which they want students to have. Then I believe the colleges should provide a good deal of help with, and encouragement in, the task of giving students in the high schools as much mathematics as possible.

At this point the third and last question becomes a matter of central importance. The college should, I believe, assure the high school teacher that her students will be given power tests and achievement tests in an effort to place them in a first course in college mathematics which will prove both challenging and possible. With an adequate placement program in the college, a few topics neglected in the high school will be of lesser importance. Moreover students who have an adequate background in high school algebra and trigonometry will not be required, as they so often are now required, to repeat these subjects upon entering college. Thus it may become possible to make analytic geometry a first college course for many students entering college. In the future one might even hope for an early introduction to calculus in college, once the high schools begin to provide a meaningful program of algebra and geometry as opposed to the dash through the mechanics of many topics which the high schools seem to feel impelled to provide at present.



Please mention the MATHEMATICS TEACHER when answering advertisements

Cinderella and the Glass Slipper; or, How Soon Is Midnight?*

By Joseph Seidlin
Alfred University, Alfred, New York

The title of my talk may seem facetious. It isn't. And what I am about to say to you is not funny.

A pseudo-civilization, because of mass stupidity, cupidity, and a distorted and misdirected selective mentality, misled itself into a scientifically and technologically superb warfare. Thus we could point with pride to the scientific and technological achievements of our period. The basic sciences became the apple of our eye. Mathematics rose to dizzy heights of indispensability. In fact, I have heard it whispered—mind you, whispered—that without mathematics there would have been no ATOMIC BOMB.

But do not let the recrudescent brilliance of the utilitarian importance of mathematics blind us to the plight of mathematics in the secondary schools. Even during the period of the Arsenal of Democracy and the early war efforts proper, when the Prince of Mechanized War discovered that the dainty foot of mathematics fitted snugly the silver slipper of the gadgets of destruction, the courses of study and the teaching of secondary school mathematics continued to be under a more or less justifiable barrage of criticism. When the A. S. T. programs were set up and the poverty of mathematical training and insight came to the surface, a loud chorus sang out that the secondary schools failed to give our boys and girls an adequate mathematical training. Some time ago, I read a detective story. I have forgotten most of it except one episode. The villain stuffed wads of cotton into his victim's mouth, then taped it ("professionally," the author said), and then, prodding him (the victim) with the butt of a gun, insisted that the victim "talk plenty and clearly."

Will teachers of secondary school mathematics ever learn that the prosperity of mathematics as an overworked handmaiden to science, does not, in itself, vouchsafe the rightful place of mathematics in our secondary schools. In fact, there is some danger that the glitter of scientific application may give a false sense of continued grandeur—even past the midnight hour.

We have won the war—with the aid of mathematics. We must continue the fight for peace—with the aid of mathematics.

Can the teachers of secondary school mathematics honestly claim that a high school curriculum which provides prominent and ample space for mathematics becomes thereby a better instrument of education in a peace-loving Democracy? What evidence will they adduce in support of their claim? What is there in or about secondary school mathematics that will be as effective in the development of a citizen of one world as, say, modern languages, Latin, sciences, social studies, or English?

We no longer need to justify the teaching of any part of mathematics which we can show to be a direct and integral part of the daily living of the uncommon common people. In fact, we find ourselves attacked from all sides—friend and enemy alike—for not teaching, or not teaching well, the obviously essential mathematics. I may say in passing that the teachers of secondary school English will find themselves before long in a tougher spot than any ever occupied by the teachers of mathematics. The question that is slowly being formed on the lips of our more thoughtful

* Address delivered before the Mathematics Section of the State Teachers Association in Rochester, New York, October 24, 1946.

citizenry is, Why does it take so much time to teach so little English? I am mentioning this merely to give you the somewhat questionable comfort that, they say, comes from the well known human trait of gregariousness (misery loves company).

During the war we had no need for justifying the teaching of any kind of mathematics. Everybody accepted the principle: "Teach them a great deal of all kinds of mathematics: some of it may come in handy some time." Well, the war is over. Teachers of secondary school mathematics had better recognize the fact that many of the war inspired objectives for teaching mathematics must be replaced by peace-inspired objectives.

Citizens of a powerful nation that must be kept as good as it is strong, look sharp to those objectives in the teaching of secondary school mathematics which, if achieved or achievable, will help our boys and girls to develop into socially effective, decent human beings. Then honestly and courageously fit courses of study and the quality of teaching to the task defined by your objectives.

The history of mathematics, as most of you know, is full of "golden ages." You are also aware of the real and spurious benefits to learners derived or derivable from the mere study of mathematics. Partly because of the very nature of the subject, partly because of the recurrently glorified and exalted position in the history of intellectual achievement that the subject has occupied, a great deal of material and interpretations of material—even in its elementary branches—assumed a sacredness not unlike the quite recently destroyed myth of the divine right of kings. I am not unmindful of the many attempts that have been made and are being made by groups and individuals to remove some atrophied or obsolete content here and there, now and then. It seems to me, however, that the time is long past for sporadic and still pretty servile hit and run tactics.

As a good many of you know, primarily I am a teacher. All teachers in this audience know what that means. As a mere human being—and Heaven knows that's "mere" enough—I derive some personal satisfaction from the fact that you listen to me attentively rather than otherwise. But as a teacher, the only real satisfaction I could possibly have from talking with you is some kind of evidence that both you and I have learned something, at least partly as a result of my talking with you. I am leading up to a concrete situation, viz. a committee composed of capable and sincere teachers and students of the teaching of secondary school mathematics in the State of New York, has been at work for some time now on a revision of our courses of study in mathematics. The committee is now seeking help and guidance and instruction in the evaluation —both through criticism and actual experimentation—of a proposed six-year syllabus in mathematics. May I suggest to you and, thus quite directly, to the Committee and the State Education Department, that before us is an opportunity which may not knock again for some time to come. As I have said to some of you at other meetings, and in other connections, the only way to improve the teaching of secondary school mathematics is throuch better teaching. In fact, I am convinced that there is no other way. Nevertheless, *what* we teach better is extremely important. Insofar as we can be certain that the materials we use are *essential* in the achieving of our objectives, our teaching will be better. This is probably as good a place as any to analyze one of our 57 varieties of slogans, viz. "Don't teach subject matter; teach children."

So far as I know, there is no acceptable definition of teaching which does not, in one way or another, bring in learning. As for me, I prefer that very harsh definition of teaching, to wit: "where there is no learning, there has been no teaching." (The converse of that is obviously not only a part of the definition.) Now it is only a small logical step to the principle that wherever learning takes place, there

must be a learner. The learner need not be a human being—it may be a dog or a horse or a cow. But a learner there must be. Therefore, when we say "teach," we imply a learner. Therefore, the injunction, "don't teach subject matter" is meaningless; "teach children" is, in this connection, redundant. A learner either learns something or learns nothing. If he learns nothing, he does not learn. If he learns, he learns *something*. Therefore, the process of teaching requires both a learner and something to learn. Now I am certain that when I get through, a good many of you will try to help me analyze the slogan, "Don't teach subject matter, teach children." It is too bad, isn't it, that even the well-schooled are just as gullible in accepting, or at least mouthing, educational slogans, as are the least schooled people in accepting, or mouthing, "commercial" slogans.

Would you be willing to accept a substitute slogan for the one we have just analyzed? "Teach children *appropriate* subject matter, appropriate to their background, experience, schooling, immediate and developable interests, immediate and deferred needs." For example, to a boy or girl 12 or 13 years old, how *appropriate* are such topics as "profit and loss," "banking," "borrowing and lending money," "bonds, stocks, investments," "insurance," and "taxation"? Again, in the everyday life of a youngster of 12 or 13, how *important* and *useful* is the triangle? We may have arrived at an opportune time to answer a question of long standing, namely, "How much synthetic geometry is good for the intellectual soul of the average boy or girl of 15?" Or, more specifically, in the case of ratio and proportion, what constitutes all the geometric applications *significant* at the tenth grade level? Or, how do we justify the need for simplifying a binomial complex fraction? I suppose in this age of psychoses we must warn our pupils of the danger of introducing an extraneous root in irrational equations.

I have touched, ever so lightly, upon some of the proposed content in the six-year syllabus. I am suggesting, not at all lightly, that you—all of you—go over that syllabus in a spirit as free from indoctrination by tradition as it is humanly possible for teachers of secondary school mathematics to be. By all means let us have a six-year sequence of mathematics. But let us see to it that the mathematics in that six-year sequence is not merely prerequisite to further mathematics; is not merely prerequisite to other sciences, engineering, and technology; but that, in the main, it shall be effective in helping boys and girls to become better citizens of a better world.

Mathematics either created, or was caught in, the stream of that kind of "scientific achievement" which has produced revolution after revolution not only in multiplying and expanding the physical welfare of man, but also in the continued expansion and development of his rational powers. Incidentally, but most significantly, man has perfected instruments for destroying man. Relatively—almost at snail's pace—has come the sometimes questionable progress in the development of man's ability to get along with man. How much of the innate values of mathematics can be diverted from accelerating physical progress to that of accelerating social progress. It is imperative that we make an effort to find out. In the race between education and catastrophe, what part will secondary school mathematics play to speed up education?

You are all familiar with the question that was sometimes expressed as poetry or at least as rhyme: "What would happen to this earth of ours and its inhabitants if, in some strange way" (and it would have to be some very strange way), "all mathematics were suddenly removed?" Personally, I was never particularly impressed by that bit of sentimental sophism. But let me be the first to start "a round" of a question which is a partial converse of that other question and which, I believe, is much more real and carries a great deal

more interpretational dynamite, to wit: "What would happen to mathematics if all the earth's inhabitants were suddenly removed?"

A Chinese friend of mine used to say, "It's difficult to save face when one loses one's head." In the race between education and catastrophe, what part will secondary school mathematics play to speed up education? The answer to that question lies partly with you and other groups like you in this State and throughout the land. But let me warn you: midnight is sooner than we think.

Logical Approach to (−a) (−b)

By V. Sites
North Side High School, Fort Wayne, Ind.

H. Van Engen's article* on this topic challenged me to send to the editor of The Mathematics Teacher, for publication, my pet explanation for $(-a)(-b)=+ab$. I have used it for many years and don't know where I picked it up. It has always seemed satisfactory to me and to my pupils. The explanation as I would give it to my class follows.

"You have learned in arithmetic that multiplication is a short method of adding like addends. For instance to multiply 3 by 2, means that 3 is to be added twice. Starting with nothing and adding 3, we get three. To this result we add 3 a second time, and get 6.

$$\begin{array}{r} 0 \\ ++3 \\ \hline +3 \\ ++3 \\ \hline +6 \end{array}$$

Hence $(+3)(+2)=+6$.

This definition of multiplication, which you have learned, is a good definition for arithmetic, where we work with positive numbers only.

In algebra we extend our number system to include negative numbers—hence we have to extend our definition of multiplication. We now define multiplication thus: Multiplication is a short method of adding like addends or subtracting like minuends. It is a short method of addition when the multiplier is positive. It is a short method of subtraction when the multiplier is negative.

$(-3)(+2)$ indicates that we are to use a short method of addition. Using the long method of addition we have

$$\begin{array}{r} 0 \\ +-3 \\ \hline -3 \\ +-3 \\ \hline -6 \end{array}$$

Hence $(-3)(+2)=-6$.

$(+3)(-2)$ indicates a short method of subtraction. The long method gives

We subtract $(+3)$ twice.

$$\begin{array}{r} 0 \\ -+3 \\ \hline -3 \\ -+3 \\ \hline -6 \end{array}$$

Hence $(+3)(-2)=-6$.

$(-3)(-2)$ indicates a short method of subtraction.

We subtract (-3) twice

$$\begin{array}{r} 0 \\ --3 \\ \hline +3 \\ --3 \\ \hline +6 \end{array}$$

Hence $(-3)(-2)=+6$.

Since (3) and (2) could just as well have been any two numbers, we can generalize as follows $(+a)(+b)=+ab$; $(+a)(-b)=-ab$; $(-a)(+b)=-ab$; $(-a)(-b)=+ab$."

After this explanation of the multiplication of signed numbers, seems rational to everyone, I, of course, give the rules for multiplying signed numbers.

* The Mathematics Teacher, April, 1947, p. 182.

Picturing Fractions

By Ann Lawlor
Foreword by Caroline Hatton Clark
New York 27, N. Y.

Foreword

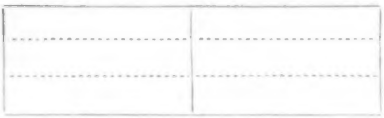
My class in Methods of Teaching Arithmetic at Child Education Foundation, New York City, is composed of students in the third year of their teacher-training course. The students have had no teaching experience except a little student teaching. Their arithmetic course is a three-strand course. One strand deals with studying and evaluating current theory and practice in the teaching of arithmetic.

A second strand places emphasis upon the necessity for teachers of arithmetic to have competence in arithmetical computation and problem solving. The students take a survey test in arithmetic at the beginning of the course. Those falling below a safe standard are assigned remedial practice materials. At the end of the course another form of the survey test is given, and credit for the course is dependent upon the student's meeting a satisfactory standard on this final testing.

The third strand of the course requires the students to make a systematic study of every rule they have ever used in arithmetic with the object of demonstrating just why the rule works. No text book is used for this part of the course. This critical analysis of rules provokes creative thinking about the number system and results in many thoughtful methods of presenting basic concepts of arithmetic. One of the students, Ann Lawlor, demonstrated how to add, subtract, multiply or divide any two given fractions with the aid of one simple drawing. I believe that many arithmetic teachers of upper elementary or junior high school level might find this presentation useful either (1) as intuitive work at the beginning of the pupil's work with fractions leading to the discovery of the formal rules for finding common denominators and performing the four fundamental operations with fractions or (2) as a summary after pupils have been taught to work with fractions to help clarify and organize their thinking. Hence I am pleased to present below a student's conception of how to picture fractions.

1. Find $\frac{1}{2}+\frac{1}{3}$
2. Find $\frac{1}{2}-\frac{1}{3}$
3. Find $\frac{1}{2}\times\frac{1}{3}$
4. Find $\frac{1}{2}\div\frac{1}{3}$

Before you solve any one of the above examples, you should be able to "see" $\frac{1}{2}$ of some whole and $\frac{1}{3}$ of that whole, because those are the fractions you are going to work with. The following steps will help you to "see" these fractions.



Step 1. Draw a rectangle to represent a whole.

Step 2. Draw a vertical line to divide the rectangle into halves. You need to think of $\frac{1}{2}$; so decide now which one of the halves of the rectangle you will work with.

Step 3. Draw horizontal lines (dotted) to divide the rectangle into thirds. You need to think of $\frac{1}{3}$; so decide now which one of the thirds of the rectangle you will work with.

Step 4. By counting you find that the rectangle is now divided into 6 equal parts. Each part is $\frac{1}{6}$ of the rectangle.

Step 5. Look carefully at the $\frac{1}{2}$ of the rectangle you decided to work with. Do you see that it equals 3 sixths of the rectangle? Look carefully at the $\frac{1}{3}$ of the rectangle you decided to work with. Do you see that it equals 2 sixths of the rectangle?

Now you are ready to find $\frac{1}{2}+\frac{1}{3}$.

$\frac{1}{2}+\frac{1}{3}=3$ sixths $+2$ sixths $=5$ sixths.

You may write the addition this way:

$$\begin{array}{r} \frac{1}{2}=\frac{3}{6} \\ +\frac{1}{3}=\frac{2}{6} \\ \hline \frac{5}{6} \end{array}$$

Now you are ready to find $\frac{1}{2}-\frac{1}{3}$.

$\frac{1}{2}-\frac{1}{3}=3$ sixths -2 sixths $=1$ sixth.

You may write the subtraction this way:

$$\begin{array}{r} \frac{1}{2}=\frac{3}{6} \\ -\frac{1}{3}=\frac{2}{6} \\ \hline \frac{1}{6} \end{array}$$

Now you are ready to find $\frac{1}{2}\times\frac{1}{3}$.

$\frac{1}{2}\times\frac{1}{3}$ means $\frac{1}{2}$ of $\frac{1}{3}$. Look carefully at the rectangle and see the $\frac{1}{3}$ you decided to work with. You want to find $\frac{1}{2}$ of the $\frac{1}{3}$, so look at the vertical line that cuts the $\frac{1}{3}$ into 2 equal parts. Each part is $\frac{1}{2}$ of $\frac{1}{3}$. But you can see that each part is $\frac{1}{6}$ of the rectangle. So $\frac{1}{2}$ of $\frac{1}{3}$ is $\frac{1}{6}$. You may write the multiplication this way. $\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$.

Now you are ready to find $\frac{1}{2}\div\frac{1}{3}$.*

$\frac{1}{2}\div\frac{1}{3}$ means how many times is $\frac{1}{3}$ contained in $\frac{1}{2}$.

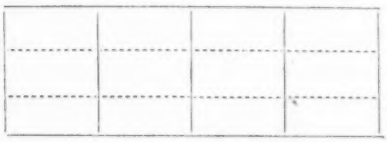
You can see that $\frac{1}{2}=3$ sixths and $\frac{1}{3}=2$ sixths; so you want to find how many times 2 *sixths* is contained in 3 *sixths*.

Look at the drawing: 2 *sixths* would fit on 3 *sixths* $1\frac{1}{2}$ times.

3 sixths $\div$ 2 sixths $=3\div 2=1\frac{1}{2}$.

You may write the division this way: $\frac{1}{2}\div\frac{1}{3}=\frac{3}{6}\div\frac{2}{6}=3\div 2=1\frac{1}{2}$.

The above presentation employed unit fractions, but the method applies equally well to multiple fractions. The accompanying diagram could be used to solve any



example involving not only $\frac{1}{3}$ and $\frac{1}{4}$, but $\frac{2}{3}$, $\frac{2}{4}$ and $\frac{3}{4}$ as well. Of course, if pupils are learning to solve examples by this method, they should construct their own diagrams themselves.

I should like to point out that pupils using this method automatically learn how to change improper fractions to mixed numbers and also how to reduce fractions to lower terms. For example, if a pupil finds that the sum of two fractions is 15 twelfths he observes on his diagram that 12 of those twelfths make a whole and that there are 3 twelfths more. In other words, he sees that $15/12=1\frac{3}{12}$. Also by observation, he learns that 3 twelfths of the rectangle is really 1 fourth of the rectangle. Hence $15/12=1\frac{3}{12}=1\frac{1}{4}$.

In conclusion, may I say that it has been a great satisfaction to me to work out a simple way to show all these fraction ideas graphically. I hope that some pupils will understand fractions better because of it.

* This graphic method of dividing fractions is helpful to pupils who have been taught or will be taught to use the meaningful "common denominator method" of dividing by fractions.

Mathematics and Sports

By Angela Pantaleone, Grade 9, Junior High School #2, Trenton, New Jersey

Have you ever stopped to think
  How math and sports are closely linked?
"Math is stupid stuff," you say,
  Yet you enjoy sports every day!

Why, basketball might not exist,
  And baseball, too, would sure be missed,
If balls were too big to be whacked,
  And courts and diamonds weren't exact.

Batting averages couldn't be found,
  They can't be expressed in figures round.
They must be calculated true,
  We see the value of math here, too.

You see math isn't "stupid stuff,"
  We really haven't space enough
To mention each and every way
  That math and sports are linked today.

Arithmetic in the Secondary School Curriculum

By W. J. LYDA

Morgan State College, Baltimore, Md.

WHILE perusing the results obtained on a written test of thirty verbal "reasoning" problems in arithmetic, the writer noted the fact that on three of the problems the number, and hence the percentage, of secondary-school pupils[1] who were average and/or above average in psychometric intelligence and who worked these problems correctly was distressingly low. Some idea of the situation can be gleaned from an examination of Table I which follows.

TABLE I

Number and Percentage of Secondary-School Pupils Who Worked Problems 25, 27, and 28 Correctly[2]

Problem	Grade Level															Total
	Seventh			Eighth			Ninth			Tenth			Eleventh			
	NC.	%	N	NC.	%	N	NC.	%	N	NC.	%	N	NC.	%	N	
25	0	0	36	1	4	28	0	0	24	0	0	10	0	0	26	124
27	1	3	36	1	4	28	0	0	24	0	0	10	0	0	26	124
28	1	3	36	1	4	28	1	4	24	0	0	10	0	0	26	124

Legend: NC. = Number of students working problem correctly.
N = Number of students.
Total = Total number of students.

These problems were taken from Child Life Arithmetic—Grade 7 and were as follows:

Problem 25: The number of games won and lost by each team in the National Baseball League during one season is given below. This table shows that Boston won 77 games out of 154 played, and that they won .500 of the total number that they played. Can you complete the table and arrange the teams in order? Figure the standings to the nearest thousandth.

Team	Won	Lost	Standing
Boston	77	77	.500
Cincinnati	80	94	—
Pittsburgh	86	88	—
Brooklyn	81	73	—
Chicago	90	64	—
Philadelphia	78	76	—
St. Louis	78	82	—

Problem 27: Mrs. Cunningham read an advertisement offering "Positive Savings" of 25% to 50% on all women's apparel. The following items were included in the advertisement:

$39.50 coats reduced to $29.75
$49.50 coats reduced to $34.75
$65.00 coats reduced to $49.75
$75.00 coats reduced to $59.75
$87.50 coats reduced to $69.75
$100.00 coats reduced to $74.75

Mrs. Cunningham figured the per cent of discount on each coat. Was the advertisement truthful?

Problem 28: On the back of the radio, Dick found the statement that it used 125 watts an hour. He estimated that the radio was in use 70 hours a month. Find the cost of operating the radio for one month at 7¢ a kilowatt hour.

[1,2] Colored secondary-school students of Augusta, Georgia, who were average and/or above average in psychometric intelligence.

The writer's concern was intensified further by certain facts pertaining to these pupils, their teachers, and the nature of the problems themselves. The pupils were familiar with the situations described in each of the problems. Their teachers were certificated in the field of mathematics. These same teachers considered these problems as realistic; that is, true to life. Presumably, the problems were seventh-grade problems. Lastly, the pupils in question were average and/or above average in psychometric intelligence.

In order to ascertain what some of the difficulties were in connection with these problems, the writer selected at random five students at each grade level from the original group of students described previously, and applied an oral technique to each student individually. The investigator asked each student to think aloud, "speak out," as he attempted to work each

problem. A record was made of what the student said and did. These records were analyzed and put in tabular form, an example of which is as follows:

TABLE II

Oral Mental Processes Employed by Five Seventh, Eighth, Ninth, Tenth, and Eleventh Grade Students—All of Whom Worked Verbal Problem 25, Incorrectly

Oral Mental Process	Grade Level 7	8	9	10	11
Read problem aloud.	5	5	5	5	5
Divide the number of games won by the number of games lost by each team.	2	—	—	—	—
Divide the number of games lost by the number of games won.	3	2	5	2	—
Subtract the number of games won by a team from the number of games lost and divide .1000 by that difference.	—	1	—	—	2
Add the number of games won and the number of games lost by each team and multiply that sum by the number of games won.	—	1	—	1	—
Add the number of games won and the number of games lost by each team and divide that sum by the number of games won by each team.	—	1	—	2	3

Upon the basis of an analysis of the oral mental processes employed by these students as well as an analysis of the written work of both the original and the limited groups of pupils, the writer noted the existence of the following arithmetical inabilities and shortages:

1. An inability to analyze a problem to see what question the problem asks, what information the problem gives, and how that information may be used to answer the question of the problem.

2. An inability to outline a method of attacking a problem by indicating the information which is necessary to solve the problem.

3. A tendency to manipulate figures without understanding.

4. A failure to check the reasonableness of an answer to a problem against the conditions of the problem.

5. A failure to do each of the following correctly:

   a. Add, subtract, multiply, and divide whole numbers, fractions, and decimals.
   b. Reduce fractions to lowest terms.
   c. Find the fractional part which one number is of another, and also per cents of discounts.
   d. Convert one denominate number into an equivalent one.

From the point of view of the writer, such results as just delineated indicate or at least are symptomatic of, the need, and desirability of more arithmetic in the secondary-school curriculum. Formal classes at the senior high school level as such are not suggested. The following recommendations may be considered as a means of improving the arithmetical competencies of high school pupils, thereby meeting a great need.

1. Periodic testing of students over the fundamental arithmetical operations.

2. Individual assignments and remedial teaching based upon the deficiencies revealed by the results of such tests.

3. Re-testing of students to determine whether or not such deficiencies have been repaired.

4. All teachers should consider themselves as teachers of arithmetic.

5. In every broad field of the high school curriculum, there are a great number of arithmetical situations and problems. The teachers in these fields should require their students to apply their knowledge of arithmetic to these problems and situations. Too, teachers should utilize as often as possible problem-solving procedures in their work.

Language Development in Mathematics through Vocabularies

By Burton F. Alexander
Petersburg High School, Petersburg, Virginia

The responsibility of developing and improving the general and technical vocabularies that are associated with elementary mathematics lies in the hand of the subject teacher. Moreover, in connection with these two principal types of vocabularies, four integral components of each should be considered, viz.: the writing, the reading, the speaking, and the potential or marginal vocabularies. A learning situation can be easily influenced by a pupil's knowledge of basic English. Vocabulary development in elementary mathematics can be influential to some degree in affecting the amount and quality of the learning outcomes.

The elementary mathematics teacher needs to recognize that:

1. Reading ability in mathematics is a major problem.
2. Language development is an actual, not artificial, growth.
3. The development of a meaningful vocabulary in mathematics is one measure of his skill as a teacher.
4. Pupils frequently use mathematical terms more or less accurately but often without the real knowledge of their correct meanings.
5. The rate of instruction is often too rapid and not adapted to all the pupils.
6. The effective means of teaching language requires the cooperation of all the teachers of the school.
7. Suitable criteria, aims, and functions should be established for the teaching of the language of mathematics.
8. Appropriate techniques should be employed.
9. Mathematics as well as other subjects in the program of the schools should increase the active vocabulary of each pupil.

Suggested techniques for vocabulary development include the use of visual aids, a variety of concrete experiences, collateral readings, contextual clues, vocabulary notebooks, synonyms and antonyms, phonetic analyses, prefixes and suffixes, configurations, dictionaries, word lists, etc. In teaching the vocabulary of mathematics, the mathematics teacher should consider those words of common discourse, the mathematical meanings of which are different from those in general use, and those words which are specifically mathematical.

Could one solution to the problem of language development in elementary mathematics be the use of vocabularies? Vocabularies have already been developed for algebra, arithmetic, plane geometry, current periodical literature, and for the seventh, eighth, and ninth grades of the junior high school. I have found no specific vocabulary for general mathematics of the eighth grade, but in a recent research study I developed a tentative vocabulary list through the cooperation of teachers and specialists in secondary school mathematics.

Vocabulary lists in elementary mathematics could prove to be excellent teaching devices. However, such lists should not be considered *must* lists. New terms should be introduced incidentally as the subject is presented. If a new term is likely to cause confusion, the criteria for selection should be carefully reconsidered. Perhaps teaching devices as these may alleviate the problems of reading and vocabulary in mathematics.

In order to evaluate eighth-grade pupils in the language of mathematics in general mathematics, I devised a language test consisting of 230 items. The test was divided into four sections: 106 multiple-choice statements and questions; 60 multiple-choice words, terms, and phrases; 13

items of recall; and 51 matching items in subsections of 7, 11, 16, and 17. The testing time of 70 minutes proved adequate, for not more than one per cent of the pupils tested failed to finish the test.

An evaluation of the testing revealed: (1) those words, terms, and phrases of general mathematics with which pupils of the eighth grade have difficulty; (2) a significant relationship between a pupil's knowledge of vocabulary and his grade in General Mathematics (The correlation coefficient was $.766 \pm .014$ for all pupils and $.757 \pm .020$ for those pupils whose I.Q.'s were known); (3) a significant relationship between a pupil's knowledge of vocabulary in general mathematics and his I.Q. (The correlation coefficient was $.669 \pm .025$); and (4) the reliability of the language test (The correlation coefficient of the odd and even items was .954).

In teaching the language of mathematics, the following criteria and aims should be considered:

Criteria

1. The language should be within the maturity, range, and understanding of all the pupils.
2. The language should be based upon actual needs and life experiences.
3. The teacher must have complete knowledge of the language of mathematics and its varied applications.
4. The vocabulary should be selected in terms of the aims and content of the course of study in mathematics.

Aims

1. To facilitate understanding, generalization, and transfer.
2. To develop in pupils a general idea of and interest in mathematics.
3. To promote accuracy, speed, good usage, spelling, and confidence.
4. To promote language growth for all the pupils.

Three important recommendations are hereby made:

1. The teacher of mathematics should also be a teacher of language usage.
2. One of the aims for every course of study in mathematics must be "the development of a meaningful vocabulary."
3. The teaching of the language of mathematics should be achieved through carefully formulated criteria, aims, functions, and techniques.

Please mention the MATHEMATICS TEACHER when answering advertisements

Directed Segments of the School Population

By VERYL SCHULT
Wilson Teachers College, Washington, D. C.

DURING the tremendous growth of secondary schools since the turn of the century, they have been suffering from unavoidable growing pains. When enrollments in high schools doubled and redoubled in decade after decade, how could the curriculum keep abreast of this growth? What could be done about keeping the mathematics offerings up-to-date? Mathematics curricula did not keep up to the changes in secondary school populations, and even today we find many high schools offering only the traditional college-preparatory sequence of the mathematics curriculum of a century ago. The study of mathematics has often been surrounded by a halo of vague superiority, or a sort of aristocracy. Some teachers have perpetuated such foolishness by trying to impress students with their own mystic ability to do mathematical tricks and to expound profound mathematical truths.

Let's look at what has happened to the curriculum. Until rather recently, algebra and geometry were required of every high school graduate. Then in 1932 the National Council Yearbook on the teaching of algebra recommended that students take algebra only if their I.Q. was above 110. But the average I.Q. of high school students in algebra courses became lower and lower as greater segments of the total population remained in the secondary school. Later, many recommendations were made that not more than 50% of the high school students should study algebra and geometry. Dr. Mallory in a talk to the Maryland mathematics teachers this past winter recommended that probably only 30% of our students should study algebra and geometry, and what the Postwar Planning Commission has called the sequential courses. In a recent issue of the California Mathematics Council Bulletin, Dr. Siemens of Compton Junior College, says, "Without question, the mathematics 'sequence' serves a very worthwhile purpose for a selected group of students who will continue their education particularly in mathematics, science, and engineering. However, the *other 85%* have an altogether different set of needs and interests in mathematics. The schools specifically, and society generally can be indicted with the charge that very little has been done to make mathematics palatable, interesting, or worthwhile to the majority of our secondary school population." That is quite a challenge, and we should face it squarely.

Some time ago, we went through a stage of oversimplifying the problem, namely guiding all those not proficient in mathematics out of the college-preparatory sequence, and offering nothing in its place. Came the war, and the need for men and women trained not in aviation, etc., but in the fundamentals of mathematics. Many high school counselors were so thoroughly impressed by the famous Nimitz letter at the beginning of the war, and other similar challenges, that every student was guided, nay, pushed, into whatever mathematics courses were offered. This often resulted either in wholesale failures or the diluting of valuable sequential courses to a disturbing degree.

The solution to the problem as I see it is, first, to offer varied mathematics courses to meet varied needs and interests, and second, to do a good job of *guidance*.

One valuable guide in setting up courses other than the sequential courses is the list of "essentials of functional competence" under Thesis I in the Second Report of the Postwar Planning Commission of the National Council of Teachers of Mathematics.* These are not the ideas

* See THE MATHEMATICS TEACHER, May, 1945.

or whims of one individual but the result of much research by the many able members of that Commission. This list is serving as an invaluable guide to curriculum workers all over the country. It is not like a study of mathematical needs of the graduates of a certain high school class, which was made by one student as a Master's thesis. The needs as described all the way through were very *meager*, but the conclusion was surprisingly significant: If the students knew more mathematics, they would use more mathematics!

The next job, after setting up courses other than the sequential, college-preparatory ones, is to make them "respectable." Is it not criminal to require young people to stay in school until a certain age and then *require* certain courses which are beyond their ability. They are the ones who when asked how they like school reply that they like it closed! Until mathematics teachers are willing to work sympathetically with the slower as well as the brighter students, adjust the work to their abilities, and treat their courses with as much dignity, we are guilty of grave injustices.

Now, suppose we have a reasonably ideal setup, with good sequential courses leading to scientific careers, and with good courses filling other needs, such as "consumer," "shop," "basic," or "practical" mathematics (whatever they may be called). How can we do a better job of GUIDING students into the most appropriate courses?

One method is through aptitude tests. Miss Ona Kraft at the meeting of the National Council in Cleveland, February 1946, reported a detailed study and splendid results of using algebra and geometry aptitude tests. We have used them in Washington to some extent and found them very valuable both in guiding students in or out of algebra and geometry, and also in grouping those who are guided into such courses. Pupil failure is gradually coming to be regarded as largely a failure of the school. Of course, there are exceptions, but often the blame lies on the inadequate program of educational guidance which the school provides.

Recently, the Head of the Guidance Department in Washington asked me if I would prepare a small pamphlet for use by the counselors, to guide pupils in mathematics. I told her that she must be psychic, because that was the very thing the Postwar Planning Commission of the National Council had been working on for the last two years, and that it was practically ready for printing. The members of the Commission had realized how much such a pamphlet was needed, and made splendid studies of the mathematics needed in various jobs and careers. They studied the mathematics needed by the farmer, the nurse, the various government workers, the engineer, the statistician, the actuary, etc.

Having collected this valuable material, the next job (and what a job!) was to write it up in language that high school students can understand readily, and short enough so that students, counselors, and teachers would take the time and effort to read it. Those of you who were at the Cleveland meeting of the Council last year recall the selections read from the different reports. Each was well written, but in a different style from every other. How was it to be integrated?

This Guidance Report as you will see is the first article in the November issue of THE MATHEMATICS TEACHER.

The Mathematics Teacher Wishes All Its Readers "A Merry Christmas and a Happy New Year"

◆ THE ART OF TEACHING ◆

Algebra Teachers—"Take It Easy"

By BROTHER B. ROBERT
The Bishop Loughlin Memorial High School, Brooklyn 5, N. Y.

EXPERIENCE has shown that some teachers encounter difficulties when working out problems at the blackboard. Yes, under the stress and strain of the teaching situation even a "natural" is susceptible to the human frailty of making mistakes at the board. Such errors lead to sad consequences: they not only demoralize the teacher, but they undermine the confidence of the students in the mathematical ability of the teacher.

An even more formidable difficulty presents itself when a disciplinary problem develops while the teacher is deeply engrossed in his board work. It is not surprising that students sometimes misbehave while the teacher is not looking directly at the class. Such conduct is to be expected.

Expenditure of energy also deserves some consideration here. Teachers who are inclined to do a tremendous amount of blackboard work might ask themselves: Are the results achieved by the teacher-centered type of lesson proportionate to the effort expended by the teacher? When the teacher handles all the board work himself the wear and tear on his vitality because of this strain is cruel. Perhaps this is a selfish way of viewing the situation, but in the long run our teaching investment will pay off more in the way of educational dividends when we are sensible about conserving our energy.

After all, why should the teacher wear himself out endeavoring to run the entire show by himself when he can readily have one of his pupils play a minor role? Why do it the "hard way" when there is an "easy way"? And that brings us to our title: "Take it easy"—*arrange to have the students do the blackboard work.*

There is no question about it, the present writer can do a better job of explaining a problem when one of the superior pupils solves the problem at the board. It stands to reason: the teacher is not attempting to do four or five things at one and the same time—watch the class, think, write, work out the mathematical calculations and explain.

Tenseness, distractions, absent-mindedness, these and other causes can and do lead to foolish miscalculations on the part of the teacher demonstrating the solution of a problem. Why should the teacher expose himself to the ridicule of the students because of these insignificant mistakes if that danger can be precluded by having one of the students take care of the problems at the board? If blunders are to be made, let a pupil make them. The teacher can thus check the error immediately.

When a student handles the board demonstration during the algebra lesson, the teacher can watch the class like a hawk during practically the entire presentation. Hence it follows that the students not only are more attentive, but have no opportunity to get out of order. It is much more satisfactory to *prevent* disciplinary problems than to attempt to *cure* them afterwards.

By enlisting the aid of a pupil, the algebra teacher can withdraw his nose from the grindstone and enjoy the broad viewpoint of the whole situation. The person who is deeply absorbed in an undertaking may be an expert at the work, but even at that, a bystander—a mere layman—can

often point out the errors of the workman or suggest some better method of doing it. As a directing spectator of the student who is demonstrating at the blackboard the teacher can lead the development of the problem step by step; he can explain the progress intelligently; he can refer to an outline if necessary; he can criticize arrangement and proportion; he can view the setup as the class sees it; and, finally, he will not as a rule leave out some step or fail to explain some significant point.

No, the student at the board does not do the teaching. The teacher is the "boss," the guiding master of the entire procedure. The demonstrating pupil is simply an auxiliary, a mechanical, silent robot who follows the teacher's instructions. By delegating this mechanical work to an assistant, the teacher conserves his energy for more important matters. The teacher is at ease, and therefore has complete mastery over himself. He will not become upset over some slight error. Neither will he be thrown out of gear, forget his outline, omit essential steps in the lesson or become a nervous wreck by inattention or disciplinary problems.

So much for the teacher. What about the pupils?

It is obvious that the role of a participant is much more interesting than that of a spectator. From the standpoint of interest it seems equally obvious that high school boys and girls derive profit from participating in the presentation of the new lesson. The superior student at the board certainly enjoys the satisfaction of demonstrating his ability to the members of the class.

But what about the other pupils? Do they get more out of this type of lesson presentation? The present writer has conducted no experiments to prove the superiority of one method over the other. It does seem reasonable to expect, though, that the fact that one of their own fellows is capable of working out a difficult problem will have a desirable psychological effect on the class as a whole—especially on those pupils who find algebra somewhat troublesome. Their line of thinking might proceed along these lines: "John can go up there and do that problem; if he can do it, so can I!"

Does the writer go over the lesson with the bright pupil before class presentation? No. Frequently this type of pupil needs no direction as to how the new principle should be handled. Perhaps he looked it up ahead of time, or maybe he just uses ordinary common sense. Otherwise, the teacher simply tells him just how to proceed if there is hesitation.

It goes without saying that the choice of students for blackboard work must be based on their ability. For our purposes pupils can be divided roughly into three groups: (1) Those with native ability in mathematics. These students are called upon to assist the teacher in presenting new material. (2) Those who learn quickly. This type is called upon to re-present material of the previous day, or to review older material. (3) Those who are average or below average. Very familiar or extremely simple work at the board is entrusted to this group.

Does the present writer ever handle the blackboard work himself? Yes, rarely. He has never attempted to present graph work in the manner discussed in this article. Sometimes, also, a student question requires a teacher answer, and then a few figures at the board to clarify the answer are more efficiently taken care of by the teacher himself. Outside of such rare occasions, this algebra teacher delegates board work to pupils, while he "takes it easy."

RENEWALS AND NEW SUBSCRIPTIONS

Please indicate whether your subscription is new or a renewal and, if possible, indicate the month of expiration.—Editor

EDITORIALS

Ina Holroyd Retires

Miss Ina Holroyd retired from active service as Professor of Mathematics at the Kansas State College of Agriculture and Applied Science at Manhattan, Kans. on July 1, 1947.

Miss Holroyd, in years of service at Kansas State, is second only to Dr. J. T. Willard, college historian. During Miss Holroyd's 48 years as professor of mathematics she has taught more than 50 of the present Kansas State faculty members. She hasn't counted up her students for ten years, but at the last tally they totaled more than 10,000. She is one of six persons who have received the college's gold medal for 25 years of faithful service.

The year Miss Holroyd was graduated there were only about 800 students at Kansas State, "and I knew almost all of them," Miss Holroyd said. She joined the staff as an assistant in the preparatory department, nine years later became an assistant in mathematics. A graduate of Kansas State Teachers College at Emporia, she has a bachelor of science degree from Kansas State college and a master of arts degree from Columbia University. She also did advanced study at Cornell University, Harvard University, and the University of Chicago. She belongs to Phi Kappa Phi, Delta Kappa Gamma, and Pi Mu Epsilon honor societies and to the Kansas Association of Teachers of Mathematics, the National Council of Teachers of Mathematics and the Mathematical Association of America.

Miss Holroyd founded the Bulletin of the Kansas Association of Teachers of Mathematics and was its editor for 18 years. This bulletin is most valuable to the teachers of Kansas, in that it forms a medium for getting information to the members of the Kansas Association of Mathematics teachers. She was on the board of directors of the National Council of Teachers of Mathematics for three years and was the Kansas representative for the National Council of Teachers of Mathematics. She was also president of the local chapter of A.A.U.W. for two years.

The Mathematics Teacher wishes to extend to Miss Holroyd its best wishes for many years of happy and useful life.—W.D.R.

Mathematics Magazine

The Mathematics Teacher is glad to announce that a magazine with a new title, "Mathematics Magazine" has just appeared to replace the "National Mathematics Magazine," founded by Professor S. T. Sanders. The first number is No. 10, Vol. XXI for Sept.-Oct. 1947. The last issue of the "National Mathematics Magazine" was Vol. XX, Nos. 1 and 2 Oct.-Nov. 1945.

The executive editorial committee consists of D. H. Hyers, University of Southern California; A. D. Michal, California Institute of Technology; Glenn James, University of California at Los Angeles. The general editorial and business correspondence will be conducted by Glenn James, University of California, Los Angeles 24, California.

There is room for another magazine of this type and The Mathematics Teacher wishes to extend its best wishes for a long and continued useful life for "Mathematics Magazine."—W.D.R.

◆ IN OTHER PERIODICALS ◆

By NATHAN LAZAR

Midwood High School, Brooklyn 10, New York

The American Mathematical Monthly

June–July 1947, Vol. 54, No. 6

1. Cairns, D. W., "Benjamin Franklin Finkel," pp. 311–312.
2. Perry, A. D., and Youngs, J. W. T., "Remarks on Analyticity and Integration," pp. 313–318.
3. Turnbull, H. W., "Colin Maclaurin," pp. 318–322.
4. Knebelman, M. S., "The Teaching of College Mathematics Today," pp. 323–326.
5. Goormaghtigh, R., "The Hervey Point of the General n-Line," pp. 327–331.
6. Mathematical Notes, pp. 332–335.
   a. Klee, Jr., V. L., "Some Remarks on Euler's Totient."
   b. Gustin, William, "Gaussian Means."
7. Classroom Notes, pp. 335–339.
   a. MacDuffee, C. C., "Objectives in Calculus."
   b. Fort, Jr., M. K., "Formula for the Area of a Triangle."
8. Elementary Problems and Solutions, pp. 339–345.
9. Advanced Problems and Solutions, pp. 345–352.
10. Recent Publications, pp. 353–354.
11. Club and Allied Activities, pp. 355–358.
12. News and Notices, pp. 358–361.
13. Official Reports and Communications of The Mathematical Association of America, pp. 361–368.

Scripta Mathematica

March–June 1947, Vol. 13, Nos. 1–2

1. Loria, Gino, "Ernest de Jonquières, Sailor and Scientist," pp. 5–15.
2. Fraenkel, Abraham A., "The Recent Controversies about the Foundation of Mathematics," pp. 17–36.
3. Lehmer, D. H., "The Tarry-Escott Problem," pp. 37–41.
4. Kasner, Edward, "Neo-Pythagorean Triangles," pp. 43–47.
5. DeCicco, John, "An Extension of Euler's Theorem of Homogeneous Functions," pp. 48–52.
6. Kasner, Edward, and DeCicco, John, "General Polar Theory," pp. 53–57.
7. Karpinski, Louis C., "Mathematics in Latin America," pp. 59–63.
8. Bradley, A. Day, "Great Circle Trigonometry," pp. 64–70.
9. Book Reviews, pp. 71–78.
10. Recreational Mathematics, pp. 15, 16, 41, 42, 47, 52, 57, 58, 79.
11. Miscellaneous Notes, pp. 119–127.
    a. Jones Philip S., "The Identity of the Author of a Hitherto Anonymous Work."
    b. Loria, Gino, "The Tragedy of a Great Astronomer."
    c. Beumer, M. G., "Gaspard Monge as a Chemist."
    d. Cowan, R. W., and Weber, B. C., "Fermat's Contribution to the Development of the Differential Calculus."
    e. Gloden, A., "Additions to Schaaf's 'List of Works on Mathematical Recreations.' "

School Science and Mathematics

May 1947, Vol. 47, No. 5.

1. Schutter, Charles H., "Improving the Quality of Instruction in Mathematics," pp. 404–408.
2. Schorling, Raleigh, "The Crisis in Science and Mathematics Teaching," pp. 413–420.
3. Gager, William A., "Computations with Approximate Numbers," pp. 424–441.
4. Price, H. Vernon, "We Can Remove the Stigma from General Mathematics," pp. 446–450.
5. Christofferson, H. C., "Teaching Directed Numbers," pp. 451–459.
6. Nyberg, Joseph A., "Notes from a Mathematics Classroom," pp. 484–487.
7. Problem Department, pp. 488–493.
8. Books and Pamphlets Received, pp. 493–495.
9. Book Reviews, pp. 495–496.

June 1947, Vol. 47, No. 6.

1. Rasmusen, Ruth B., "An Application of Degenerate Conics," p. 502.
2. Carnahan, Walter H., "Times Curves," pp. 507–511.
3. Tucker, A. N., "An Odd Locus," pp. 512–514.
4. Anning, Norman, "Is It Hero's Formula?" p. 514.
5. Young, P. M., "Nomograms," pp. 521–525.
6. Lehmann, Paul W., "A Math Circus," pp. 560–562.
7. Ignatia, Sister M., "Applied Mathematics in General Chemistry," pp. 563–568.
8. Hawkins, G. E., "Mathematics in the Modern School Program," pp. 569–571.
9. Problem Department, pp. 578–583.
10. Books and Pamphlets Received, pp. 583–585.
11. Book Reviews, pp. 585–593.

OFFICIAL NOTICE

As secretary of the National Council of Teachers of Mathematics, I officially announce the annual election of certain officers of the National Council, said election to take place February, 1948.

At the Atlantic City meeting February 26, 1938, the Nominating Committee, consisting of the two most recent ex-presidents and the secretary as chairman (for the year 1947: Rolland R. Smith, F. Lynwood Wren and Edwin W. Schreiber), was instructed to prepare an official ballot naming two eligible candidates for each elective office, reserving a blank space for a third prospective candidate whose name may be written in by the voter. The officers to be elected are: President, 1948–1949, Second Vice-President, 1948–1949, and three Directors, 1948–1950. The official post card ballot will be sent through the mail about February 15, 1948.

The periods of service of the officers of the National Council, from its organization in February, 1920, to the present time are printed on the following page.

EDWIN W. SCHREIBER, *Secretary*

The National Council of Teachers of Mathematics

Organized 1920—Incorporated 1928

*H. E. Slaught, Chicago, Ill., 1936–1937, *Honorary President*

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24. Edwin G. Olds, Pittsburgh, Pa., 1943–1944
25. Edith Woolsey, Minneapolis, Minn., 1944–1945
26. L. H. Whitcraft, Muncie, Ind., 1945–1946
27. H. W. Charlesworth, Denver, Colo., 1946–1947
28. E. H. C. Hildebrandt, Evanston, Ill., 1947–1948

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NEWS NOTES

The Women's Mathematics Club of Chicago and Vicinity holds its first meeting on Oct. 4, 1947.

Miss Haertel, President of the Club in 1939, has just returned from Japan, where she spent a year teaching the Occupation Forces. She told us of her experiences there and showed some of the motion pictures she took. These pictures were taken in Kobe, where she was stationed, as well as in several other places (including Hiroshima) she visited. Many of us remember her excellent talk on Education in Germany, telling of some work she did in that country with Columbia University.

President, Miss Edith Levin, Englewood High School.
Vice-President, Mrs. Mary Werkman, Parker High School.
Secretary, Miss Marie Brennecke, Washington School.
Treasurer, Miss Edith Inks, Oak Park High School.
Program Chairman, Miss Bernice Von Horne, Hyde Park High School.

A workshop for Teachers of Mathematics was held on August 18–23, 1947, by the Galesburg Division of the University of Illinois at Galesburg, Ill., under the direction of Daniel W. Snader.

MONDAY—AUGUST 18

Morning Session

9:00 A Word of Welcome. Introduction of the Workshop Staff.
The Professional Needs of Secondary School Teachers of Mathematics—Dr. Daniel W. Snader, Director of the Workshop Activities.
The Organization of Mathematics Study Groups*—Dr. William Betz, Author, Lecturer, and Specialist in Mathematics for the Public Schools, Rochester, New York.
Registration of Workshop Participants—Miss Martha Hildebrandt, Mathematics Department Head, Proviso Township High School, Maywood.

Afternoon Sessions

1:30–4:00 First Meetings of Study Groups.
Group 1. Dr. Betz
Group 2. Dr. Snader
Group 3. Miss Hildebrandt

TUESDAY—AUGUST 19

Morning Session

10:00 The Mathematics Used in Automotive Design (Illustrated with films, charts, etc.)—Dr. J. O. Almen, Head of Mechanical Engineering 1, Research Laboratories, General Motors Company, Detroit, Michigan.
Discussion.

Afternoon Sessions

1:00–4:00 Second Meetings of Study Groups.

Evening Session

University Theater

7:45 Mathematics in Naval Training Program—Commodore James E. Arnold, USNR, 9th Naval District. Lieutenant Richard P. Bailey, USNR, Naval Academy, Annapolis, Maryland.
Discussion.

WEDNESDAY, AUGUST 20

Morning Session

10:00 Mathematics used in General Electric Co. (Illustrated)

Afternoon Sessions

Third Meeting of Study Groups.

Evening Session

8:00 Address: Dr. Wm. Betz.

THURSDAY, AUGUST 2

Morning Session

University Theater

10:00 Making and Using Films, Instruments and Models (Illustrated Lecture)—Mr. W. Roger Zinn, Educational Consultant for Jam Handy Films, Detroit, Michigan.
Discussion.

Afternoon Sessions

1:00–4:00 Fourth Meetings of Study Groups.

Evening Session

University Theater

8:00 Application of Large Scale Computing to Scientific Problems—Dr. H. R. J. Grosch, International Business Machines Corporation, New York, New York.
Discussion.

FRIDAY, AUGUST 22

Morning Session

10:00 The Mathematics of Radio and Radar—Dr. Victor J. Young, The Sperry Gyroscope Co.
Discussion.

Afternoon Sessions

1:00 Planning a Graduate Program for Teachers of Junior and Senior High School Mathematics—Dr. Wm. Betz.
Discussion.

3:00 Study Group Reports—Miss Hildebrandt and Miss Nuess.
Dr. Daniel W. Snader, Chairman of Mathematics Department, Undergraduate Division, University of Illinois, Galesburg, Director of the Workshop
Dr. William Betz, Specialist in Mathematics for the Public Schools, Rochester, New York
Miss Martha Hildebrandt, Head of the Department of Mathematics, Proviso Township High School, Maywood, Illinois
Miss Vivian R. Neuss, Instructor in Mathematics, University of Illinois, Urbana, Illinois

Dr. J. O. Almen, Head of Mechanical Engineering 1, Research Laboratories, General Motors Company, Detroit, Michigan
Commodore James E. Arnold, USNR, Commander of the 9th Naval District, Great Lakes, Illinois
Lieutenant Richard P. Bailey, USNR, Associate Professor of Mathematics, Naval Academy, Annapolis, Maryland
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Dr. H. Poritsky, Engineering General Division, General Electric Company, Schenectady, New York
Dr. Victor J. Young, The Sperry Gyroscope Company, Inc., New York, New York
Mr. W. Roger Zinn, Educational Consultant for Jam Handy Films, Detroit, Michigan

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The Annual Spring Meeting of the Eastern Division of the Colorado Education Association, Mathematics Section, was held on April 26, 1947 at Denver, Colorado.

The first part of the program was devoted to the theme, "Frustrations of a Mathematics Teacher," and Miss Sarah B. Duboff of the University Park School, Denver, spoke from the standpoint of an elementary teacher, while Mr. George A. Cutting of Cole Junior High School, Denver, gave views of a junior high school teacher.

In the second part of the program Dr. Aubrey J. Kempner of the University of Colorado, Boulder, read a paper on "Theory of Numbers."

Members of the Executive Council: President, Kenneth E. Gorsline, 1451 Grape St., Denver; Vice-President, Prof. Aubrey Kempner, Univ. of Colo., Boulder; Secretary-Treasurer, Burnett Severson, 2755 Steele St., Denver.

Term Expires 1947: Grace Kenehan, Denver; A. J. Lewis, Denver; Irene Mott, Denver; Ruth L. Roberts, Denver.

Term Expires 1948: Russell Casement, Denver; Leota Hayward, Fort Collins; Ruth Irene Hoffman, Denver; Helen Taylor, Denver.

Term Expires 1949: Ethel Gordon, Denver; Dwight Gunder, Fort Collins; Margaret McGinley, Denver; Dorothea Williams, Denver.

The first meeting of the Men's Mathematics Club of Chicago and Metropolitan area (Organized in 1913) was held on Friday Oct. 14, 1947. Professor Mason E. Wescott of Northwestern University spoke on the topic "The Use of Statistics for Controlling Quality in Manufacturing Processes."

Professor Wescott brought along several demonstration devises which have been used in teaching quality control methods to men in industry. He is at present Editor-in-Chief of "Industrial Quality Control," the official publication of the American Society for Quality Control, Inc.

W. G. Hendershot, Secretary-Treasurer.

Frank B. Allen, Chairman, Program Committee.

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A conference on the Teaching of Secondary School Mathematics was sponsored by Northwestern University at Evanston, Illinois, on Saturday, May 3, 1947.

First General Session
Auditorium, Technological Institute
9:30 A.M.

Chairman: Dean Fred D. Fagg
Address: Functional Competence in Mathematics
Dr. William Betz, Specialist in Mathematics, Public Schools, Rochester, New York

Discussion Group Meetings
10:45–12:15

Group I, Room 108: What are the Mathematical Ingredients of the Curriculum in Algebra?
Leader: Joseph J. Urbancek, Chicago Teachers College
Group II, Room 162: What are the Mathematical Ingredients of the Curriculum in Geometry?
Leader: Walter Barczewski, Waukegan Township Secondary Schools
Group III, Room 167: What are the Applications of Mathematics that we can emphasize in our Teaching of Algebra and General Mathematics?
Leader: Virginia Terhune, Proviso Township High School
Group IV, Room 274: What are the Applications of Mathematics that we can emphasize in our Teaching of Geometry?
Leader: Florence B. Young, Steinmetz High School
Group V, Room 181: What are the up-to-date Learning Processes involved in the Mastery of Mathematics and its Applications?
Leader: George E. Hawkins, Lyons Township High School and Junior College
Group VI, Room 213: What are the Lesson Types in Algebra that the Teacher should know?
Leader: Francis W. Rungo, Evanston Township High School
Group VII, Room 222: What are the Lesson Types in Geometry that the Teacher should know?
Leader: Milton D. Oestricher, Francis W. Parker School, Chicago
Group VIII, Room 223: What are the latest Psychological Aspects of Transfer?
Leader: Richard E. Gadske, New Trier Township High School
Group IX, Room 271: What is the Role of

Practice (Drill) in Mathematics Teaching?
Leader: Elsie Parker Johnson, Oak Park Township High School
Group X, Room 175: What is the Psychology of Conceptual Thinking?
Leader: Glenn F. Hewitt, Von Steuben High School
Group XI, Room 305S: What are the Modes of Testing for Functional Competence?
Leader: Eugene W. Hellmich, DeKalb State Teachers College

Second General Session
Auditorium, Technological Institute
1:45 P.M.

Chairman: Dean J. M. Hughes
Summary of Discussion Group Meetings, presented by Leaders of each Discussion Group
Question-Answer Period—Dr. William Betz
Summary of Conference—Dr. Betz

◆ BOOK REVIEWS ◆

Mathematics as a Culture Clue. By Cassius Jackson Keyser. And other essays. Scripta Mathematica, Amsterdam Ave. and 186th St., N. Y. vii+277 pages. Price $3.75.

This book is a collection of essays and addresses given by the author during the middle thirties. To those unacquainted with the penetrating works of Professor Keyser, as exhibited in his *Mathematical Philosophy* and *Pastures of Wonder*, and who have not the time to spend in much study, this book offers an excellent introduction. The essays are written in simpler style and are easier to read than his original books. In general they cover three areas: 1, a review of the author's writings; 2, a review of current philosophical writings; 3, a review of several outstanding scientific contributions and their authors.

The essay which is also the title of the book is a summary of Oswald Spengler's philosophy as exhibited in "The Decline of the West." Keyser concurs in Spengler's thesis on the growth and decay of civilizations and cultures, especially as related by the part mathematics has played in the various cultures. One can find here additional evidence in support of Spengler's view and also a more vivid picture of the relation of mathematics to the other fields of knowledge in each of the several cultures. In a similar fashion, the reader is introduced into Count Korzybski's *Science and Sanity* in the essay titled "Mathematics and the Science of Semantics."

In the essays reviewing his own work, Keyser gives a very simple and clear picture of the difference between pure mathematics and applied mathematics, and between a proposition and propositional function. The treatment is extended to the explanation of Doctrinal Function and an abstract system of all knowledge for which the author has coined the name *Panthetics* or universal systems.

Anyone who has attempted to master the logic and philosophy of Charles Saunders Peirce will appreciate the review of the first five volumes of *Peirce's Collected Papers*. Keyser succeeds by extracting a large number of quotations, along with his own pertinent interpretation, in giving a comprehensive and appreciative picture of the breadth, depth, and vision of this philosopher and logician. Other essays introduce the reader to some of the scientific contributions of William Benjamin Smith, Velfredo Pareto, and Havelock Ellis.

The author is a renowned teacher of mathematics. Like any true teacher he wishes to share his knowledge and thoughts with others. It appears to this reviewer that the essays and addresses are the result of this urge. Having seriously pored through the five volumes of Peirce's work, Keyser had to tell his impression and interpretation to all those who did not have his leisure for such study, and thus his essay *Charles Saunders Peirce*. And so with the other essays. The writing is in typical Keyser style, scholarly in vocabulary and expression, and "Time-exploring, space-exploring and mind-exploring." Teachers and friends of mathematics should find this collection a handy reference and satisfying occasional reading.—HOWARD F. FEHR.

A Handbook on Curves and Their Properties. By Robert C. Yates. J. W. Edwards, Ann Arbor, 1947. x+245 pages. Price $3.25.

This book was originally prepared by Lt. Col. Yates exclusively for the use of the Mathematics Department of the U. S. Military Academy. The interest and approval of the mathematicians who were privileged to see the first edition have led to the publication of this second edition for general sale. In this edition all the errors noted by the reviewer in the first edition have been corrected, the discussions of some curves have been expanded as have also the cross-references and bibliographical notes.

As its title indicates, the book is a handbook of plane curves. It is arranged alphabetically from "Astroid" to "Witch of Agnesi" with

cross-references within the text and an index added to substantially increase its usefulness. The general organization includes for each curve or family of curves a sketch of its history, a discussion of one at least and usually several ways of describing or generating the curve, various forms of its equation, including its Whewell and Cesaro intrinsic equations, a summary of its metrical properties such as its length, area, radius of curvature, the area and volume of the one or more surfaces of revolution which it may generate, and finally a list of general items which often include some of its applications and its relationships with other curves. Interesting linkages and mechanical devices for constructing the curves are frequently described and constructions for tangents and normals are often given which make use of the instantaneous centers of rotation of such devices.

In short, this book is a veritable mine of interesting enrichment material that every teacher of analytic geometry and sponsor of a high school or college mathematics club will find of great and continuing use. This is a teacher's handbook for which many of us have long wished. It contains many of the facts which, in these days of less emphasis on advanced work in algebraic geometry and higher plane curves, are never met by graduate students in mathematics, and the lack of which is felt by them only when they in turn come to teach and to seek to add interest and perception of interrelationships to their own presentations.

The omission of simple derivations and explanations emphasizes its primary function as a teacher's handbook; however, the author in the preface does outline a selection and a sequence of topics which could be used in constructing a college course. Many of the curves and their applications could be used with older and superior high school students for enrichment material, projects, and club programs. The high school teacher himself might find much of personal interest as well as a broadening of his own view of the whole of mathematics through browsing in the book.

The first criticism of any reader who has felt the need of such a handbook and has tried to do something about it himself has been anticipated by Dr. Yates. That is, Yates has occasionally left room in the text where the reader may add his own notes, references, and additional properties. It has been necessary to select only the most significant ideas and to condense their discussion in a book which is to encompass so many curves in a limited space. This Yates has done very well, but any interested reader is sure to have his own treasured ideas which he will want to insert.

There are only a few places where the reviewer feels that the purpose of the book would dictate the addition of a few comments. For example, with regard to the conics, the most commonly studied of the curves treated, it would seem that the name of the Belgian, G. P. Dandelin, and the date 1822, should have been associated with the spheres in the cone on pages 37 and 38 to illustrate the fact that though the conics originated with the Greeks circa 350 B.C. new facts about them have continued to be discovered until modern times. Likewise merely to note that the conics were connected with the famous Greek problems and not to display the solution of the trisection problem by the use of the hyperbola seems an oversight in a teacher's handbook (especially since Yates himself earlier wrote a very nice book on *The Trisection Problem*), and certainly the bibliography on conics should refer to J. L. Collidge's *A History of the Conic Sections and Quadric Surfaces*.

At first glance $3.25 may seem to be a high price for a lithoprinted book, but in addition to the fact that all prices are up are the facts that the printing and the many (211) diagrams are uniformly neat and clear, and that the permanent stiff binding is well made and good looking. These facts coupled with the wealth of ideas in and general usefulness of the book lead the reviewer to recommend it unhesitatingly as a worthwhile addition to any teacher's library. —PHILLIP S. JONES.

Guidance Pamphlet in Mathematics

THE MATHEMATICS TEACHER is already being swamped with orders for reprints of the Guidance Report of the Commission on Post-War Plans in Mathematics which appeared in the November issue of the TEACHER. We are also receiving many compliments about the Report. Professor Odom of Louisiana Polytechnic Institute said the November issue alone is worth the entire year's subscription. Benjamin Braverman of Seward Park High School, New York City, said that the Report should be made available to every pupil entering the ninth year. Because of the interest in the Report and its value to teachers and pupils alike THE MATHEMATICS TEACHER is reprinting the Report with covers and will sell them in quantities of 10 or more at 10¢ each postpaid. Mathematics Clubs, State association groups of mathematics teachers and others will no doubt wish to order the Report in quantities and distribute them where they will do the most good.—EDITOR.

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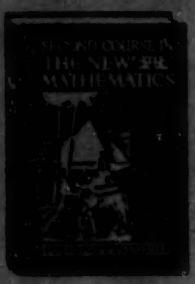
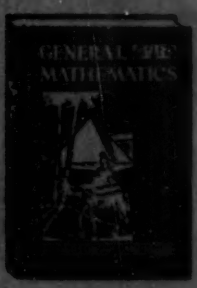

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